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Discrete reliability growth models using failure discounting

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THESIS

DISCRETE RELIABILITY GROWTH MODELS
USING FAILURE DISCOUNTING

by

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Discrete Reliability Growth Models
Using Failure Discounting

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requirements for the degree of

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ABSTRACT

Three discrete reliability growth models using fractional failure reduction, referred to as failure discounting, were developed to estimate changing system reliability. Each of the models is designed for use when testing is performed until a fixed number of failures have been observed and attribute data, success or failure, is available for each trial. The first reliability growth model applies failure discounting to the maximum likelihood estimate for a proportion. The second and third models use a modification of an exponential reliability estimate employing linear regression and a weighted average technique respectively along with failure discounting to track changing reliability. Two failure discounting methods were used with each reliability growth model. The first method reduces past failures by a fixed fraction at a fixed interval. The second method uses the upper confidence bound for the reoccurrence of each failure cause as the discounted failure value. The performance of the reliability growth models with varying reliability growth patterns was evaluated with a Monte-Carlo simulation.

THESIS DISCLAIMER

The reader is cautioned that computer programs developed in this research may not have been exercised for all cases of interest. While every effort has been made, within the time available, to ensure that the programs are free of computational and logic errors, they cannot be considered validated. Any application of these programs without additional verification is at the risk of the user.

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I. INTRODUCTION

Accurately determining the reliability of a newly designed piece of hardware is an important part of material acquisition in both the military and civilian sectors. Significant amounts of both time and money are spent testing new systems to determine if they meet specified reliability requirements. Since most organizations are under constant pressure to conserve both time and funds, it is important to develop methods of estimating system reliability at minimum cost. When conventional estimators for system reliability are used, verifying high reliability requires a great deal of test data costing both time and money. This problem is compounded by technological advances which produce more complex and highly reliable hardware at an ever increasing cost. Developing new and more accurate methods of estimating reliability that reduce required testing could result in significant savings of resources.

Early versions of newly designed equipment often exhibit low reliability. As engineering designs are tested and improved the system reliability usually grows rapidly. This process continues throughout development phases and into field use. By the time a product design reaches its final configuration, a large amount of test data is usually available from tests performed on previous configurations, subcomponents and parts. Since conventional estimators assume a constant reliability, it is necessary to wait until the finished piece of hardware is available to begin reliability verification. This practice is not an efficient use of constrained resources because results from previous testing are discarded. If all the data available could be used to estimate final configuration reliability, the lead time and cost of the finished product could be reduced. Reliability growth models serve this purpose. They use data from testing at all phases of development to establish a reliability growth pattern. This growth pattern can then be used to provide current estimates of system reliability with less testing than standard testing programs.

Reliability growth models can be classified into two categories, discrete and continuous models. This paper will address discrete reliability growth models that employ failure discounting. These growth models are used to determine reliability for systems where test results can only be stated in terms of success or failure, ie; attribute data. Failure discounting allows the user to fractionally discount past failures when

resultant corrections to equipment are verified through subsequent testing, as actual corrections to the cause of failure. The use of failure discounting makes it possible for previously linear estimators to track a non-linear growth pattern.

The objective of this paper is to develop and evaluate the performance of discrete reliability growth models using failure discounting. Three different reliability growth models are proposed and evaluated with two different methods for discounting past failures. Monte-Carlo simulation is used to evaluate each of the three growth models under both discounting methods. The simulation uses actual reliabilities to generate a random growth pattern. From this growth pattern, test data is then generated for the known system reliability at each testing phase. Each of the reliability growth models can then be applied to the test data producing an estimated growth pattern. The performance of each reliability growth model is evaluated by comparing the estimated growth pattern with the actual reliability in each phase.

The following chapter will address each of the three reliability growth models used and the two methods of failure discounting employed. Chapter III contains a discussion of the computer simulation used to evaluate each of the models. Chapter IV presents the results of the evaluation and Chapter V summarizes the study stating conclusions and recomendations for future study.

II. DISCRETE RELIABILITY GROWTH MODELS

A. BACKGROUND

Three different reliability growth models and two different techniques for fractionally discounting past failures are discussed in this chapter. The reliability growth models can be divided into two categories. Models using a modification of the maximum likelihood estimate, MLE, for a proportion, referred to as the MLE With Discounting Model, a model referred to as the Weighted Average Model and an Exponential Regression Model. All of the models may be applied with or without failure discounting and are independent of the discounting method.

Two different methods for fractionally discounting past failures are also discussed. The first method is referred to as the Standard or Straight Percent Discounting Method. The second was proposed by David K. Lloyd and will be referred to as the Lloyd Method. Before the specific details of each growth model and discounting method are discussed, several terms need clarification.

Reliability, R, as referred to in this study is defined as the probability of a system successfully completing a single trial or operation cycle, Equation 2.1 . $\underline{R}$ will be used to denote an estimate of the actual reliability R.

$$R = \Pr(\text{ SINGLE TRIAL SUCCESS }) \qquad \text{(eqn 2.1)}$$

Discrete reliability growth models are normally used for trials which can only be evaluated in terms of a binary variable, success or failure, i.e. attribute data. Exactly what constitutes a trial is normally defined by the test agency but could include, for example, a single rocket engine ignition attempt or a single test firing of a missile up to and including target destruction. To use the reliability growth models discussed here, trials must be defined such that they are discrete and can be evaluated as a success or a failure.

A testing phase is defined as a collection of trials, one or more, for which the configuration of the system being tested is unchanged. This implies that the actual system reliability remains constant throughout a single phase. If a test-fix-test methodology is employed, design changes or "fixes" are implemented after each failure.

In this case, a phase will consist of one failed trial and all the successful trials leading up to the single failure. If on the other hand a test-find-test methodology is used, a single phase may include numerous failures. Under both methodologies, a phase is the collection of all trials between configuration changes. [Ref. 1]

Because the underlying reliability of the system may be different from phase to phase, a new estimate of reliability must be computed for each and every phase. As such, R_k ,denoting system reliability and $\underline{R}_k$ denoting the estimate of system reliability in phase k, will be used throughout the paper.

Annotating the cause of each failure is critical to all discounting methods considered in this study. How the failure cause is defined, is dependent upon the judgement of the model's user and the type of system to which the model is applied. If the model is used to determine the reliability of a single component, failure cause refers to the reason for failure such as a broken cam-shaft or fuel stoppage. If the model is applied to determine the reliability of a system of components, the failure cause may refer to the failure of a sub-component or the specific reason for failure. To apply the discounting methods discussed in the paper, the model user must determine if two observed failures are due to the same cause. It will not be necessary, however, to enumerate all possible failure causes. The level of detail at which failure causes are defined is up to the user and should be determined before any testing is begun. The discounting methods studied will be discussed in the next section.

B. DISCOUNTING METHODOLOGY

Weaknesses in a system design become known as they cause failures during testing. As weaknesses are identified and corrective "fixes" are implemented the probability of a repeat failure due to the corrected cause should be reduced and system reliability improved. This concept forms the basis for failure discounting. Once a system weakness has been corrected and the improvement validated through further testing, fractionally discounting the past failure is a method of reflecting the improved system reliability in the previously collected data. When using standard reliability estimators such as the maximum likelihood estimate, Equation 2.2

$$\underline{R} = \frac{\text{Number of Successful Trials}}{\text{Total Number of Trials}} \qquad \text{(eqn 2.2)}$$

only the data from the phase of interest can be used to determine system reliability because system reliability is different from phase to phase. If the proper discounting methodology can be developed, failure data from all past phases could be discounted and made compatible with the data from the current phase. Increasing the amount of data available for computing the estimate of current system reliability will increase the accuracy of the estimate and reduce the amount of testing necessary to verify system reliability.

Of equal importance to a method for discounting past failures is a penalty system that will restore all or some portion of a previously discounted failure if its failure cause should reoccur. If a previously implemented "fix" proves not to have corrected a design weakness, the discounts applied to its failures will diminished - erased reversing the effect of discounting. A properly designed penalty factor will make the discounting method self-correcting.

In order for a discounting method to be useful, it must be flexible enough to handle many varied reliability growth patterns. To achieve this flexibility, discounting methods with one or more input parameters are proposed. Regardless which discounting method is used, experience and good engineering judgement will be necessary to choose input parameters that are compatible with both the reliability estimator used and the proposed test plan. If improper parameters are applied, the model may consistently over or underestimate system reliability. The following sections describe the specific steps for applying two failure discounting methods including examples of their use.

1. Straight Percent Discounting Method

The Straight Percent Discounting Method removes a fixed fraction of a failure each time a predetermined number of trials are observed without reoccurrence of the failure cause. Before a failure is discounted, the design "fix" correcting the failure cause should be verified by further testing. This concept forms the basis for the discounting method. The fraction of a failure removed, F, is referred to as the discount fraction. After the initial occurrence of a particular failure cause, any trial in which the failure cause is not repeated is considered a success. The number of successful trials between application of the discount fraction is referred to as the discount interval, N. Both N and F are input parameters specified by the user. They remain constant throughout failure discounting computations. To use the Straight Percent Discounting Method, it is necessary to record the number of successful trials since the last occurrence of each

observed failure cause, referred to as M. Failure discounting is performed at the end of each testing phase using the values for N, F and M.

The mathematical representation of the straight percent discounting method is shown in Equation 2.3

$$\text{ADJUSTED FAILURE} = (1 - F)^{\text{INT}(M/N)} \qquad \text{(eqn 2.3)}$$

F = Fraction of a Failure to be Removed

M = Number of Successful Trials Since Last Failure for the Failure Cause

N = The Discount Interval Expressed in the Number of Successful Trials

INT is a function which returns the truncated integer portion of its argument. M is set to zero at each occurrence or reoccurrence of a particular failure cause. This causes the value for adjusted failure to be set at 1.0 erasing any previously applied discounting. This also functions as a penalty factor allowing the model to correct itself should past discounting prove unwarranted. By altering the choice of N and F the amount of discounting applied can be varied from none to total loss of a failure. This allows the method to be applied in situations verying from little or no reliability growth to rapid reliability growth.

An example will help clarify the use of the method. Consider the test results included in Table 1. The data displayed in the table is from a test-fix-test program in time order sequence. A design fix was applied to the equipment after each failure. As such, the actual system reliability does not remain constant. The outcome of each trial was generated from the geometric distribution using the actual reliability listed in the table. The cause for each failure was generated using a stochastic process and has been coded using letters. No more information concerning failure cause is required because the data are used to determine if the current failure cause has occurred previously.

To apply Straight Percent Discounting the two input parameters must be determined. For the example

N = 3

F = .25

will be used. The Straight Percent Discounting Method is applied sequentially, failure by failure, recomputing M, the number of successful trials since last failure, for each failure cause as the next data entry is included. At the end of each testing phase, the

TABLE 1
SAMPLE TEST RESULTS

TRIAL NUMBER	FAILURE NUMBER	FAILURE CAUSE A	B	C	D	RELIABILITY
1		S	S	S	S	.358
2	1	F	S	S	S	.358
3	2	S	F	S	S	.412
4	3	S	S	F	S	.518
5	4	S	S	S	F	.596
6		S	S	S	S	.627
7	5	S	S	F	S	.627
8		S	S	S	S	.721
9		S	S	S	S	.721
10	6	S	F	S	S	.721
11		S	S	S	S	.780
12		S	S	S	S	.780
13		S	S	S	S	.780
14		S	S	S	S	.780
15		S	S	S	S	.780
16		S	S	S	S	.780
17		S	S	S	S	.780
18	7	F	S	S	S	.780

S : DENOTES SUCCESS
F : DENOTES FAILURE

value of M, along with the input parameters F and N, is used in Equation 2.3 to determine the adjusted failure value. This procedure is applied to the data in Table 1.

When the sixth failure is included, all failures due to Causes A, C or D have values for M greater than or equal to three causing the discount fraction to be applied. Computation of the adjusted failure values for Cause A is illustrated in the following calculations.

$$\text{ADJUSTED FAILURE} = (1 - .25)^{\text{INT}(8/3)}$$
$$= (.75)^2$$
$$= .5625$$

Failure two and failure six were both due to Cause B which resets their corresponding M values to zero and their adjusted failure value to unity. The results for all failures to date are shown below.

FAILURE NUMBER	M	FAILURE CAUSE	ADJUSTED FAILURE
1	8	CAUSE A	.5625
2	0	CAUSE B	1.0000
3	3	CAUSE C	.7500
4	5	CAUSE D	.7500
5	3	CAUSE C	.7500
6	0	CAUSE B	1.0000

Finally, failure seven is included and the failure discounting computations are completed. The number of trials, eight, causes further discounting for all previous failures except for the first failure which is also due to Cause A. The final results of applying the straight percent discount method to the data in Table 1 appears in Table 2. The values in the adjusted failure column are the current values of each fractional failure after trial number eighteen has been included. For example, the two adjusted failure values due to Cause B have recorded eight successfull trials since the last failure due to Cause B and INT (8/3) = 2. Therefore, the adjusted failure value equals $(.75)^2 = .5625$. The remaining adjusted failures are computed in the same manner yielding the fractional failure values after-eighteen trials and seven failures.

2. **Lloyd Failure Discounting Method**

The underlying premise of the Lloyd Discounting Method states that the fraction of a failure removed by the discounting method should not be arbitrarily chosen. Some statistical basis should be used to determine how much, if any, a past failure should be discounted. Lloyd proposes using the upper confidence limit for the probability that a failure cause will reoccur as the discounted value of its corresponding failure. This technique allows the user to control the amount failures are discounted by

TABLE 2

STRAIGHT PERCENT DISCOUNTING
OF EXAMPLE DATA (TABLE 1)

TRIAL NUMBER	FAILURE NUMBER	FAILURE CAUSE A	B	C	D	ADJUSTED FAILURE
1		S	S	S	S	
2	1	F	S	S	S	1.0000
3	2	S	F	S	S	.5625
4	3	S	S	F	S	.4219
5	4	S	S	S	F	.3164
6		S	S	S	S	
7	5	S	S	F	S	.4219
8		S	S	S	S	
9		S	S	S	S	
10	6	S	F	S	S	.5625
11		S	S	S	S	
12		S	S	S	S	
13		S	S	S	S	
14		S	S	S	S	
15		S	S	S	S	
16		S	S	S	S	
17		S	S	S	S	
18	7	F	S	S	S	1.0000

S : DENOTES SUCCESS
F : DENOTES FAILURE

specifying the level of confidence for the confidence bound. As such, a single input parameter, the confidence level γ, is required to perform failure discounting. [Ref. 2]

The discounting equation is derived from the upper confidence limit for the probability of failure due to a particular failure cause. This confidence interval is based on the number of successful trials recorded since implementation of the "fix" for the failure cause. Assuming the "fix" is implemented immediately after failure, this will be equivalent to the number of successful trials since last failure, previously defined as M.

$$
\begin{aligned}
Q &= \text{PROBABILITY OF FAILURE IN A SINGLE TRIAL} \\
&= 1 - R \\
\gamma &= \text{FRACTIONAL CONFIDENCE LEVEL} \\
M &= \text{NUMBER OF SUCCESSFUL TRIALS SINCE LAST FAILURE} \\
\underline{R}_{L(\gamma)} &= \text{LOWER } \gamma\% \text{ CONFIDENCE LIMIT FOR RELIABILITY}
\end{aligned}
$$

$$
\begin{aligned}
\underline{R}_{L(\gamma)} &= (1 - \gamma)^{1/M} \\
\gamma &= \Pr[\, R \geq \underline{R}_{L(\gamma)} \,] \\
&= \Pr[\, 1 - R \leq 1 - \underline{R}_{L(\gamma)} \,] \\
&= \Pr[\, Q \leq 1 - \underline{R}_{L(\gamma)} \,]
\end{aligned}
$$

$$\therefore \text{ UPPER CONFIDENCE LIMIT for } Q = 1 - (1 - \gamma)^{1/M} \qquad \text{(eqn 2.4)}$$

$$
\begin{aligned}
\text{ADJUSTED FAILURE} &= 1 - (1 - \gamma)^{1/M} && \text{for } M > 0 \qquad \text{(eqn 2.5)} \\
&= 1.0 && \text{for } M = 0
\end{aligned}
$$

The failure discounting equation is derived in Equations 2.4 and 2.5. Because the derived equation for adjusted failures is not defined for M equal zero, the adjusted failure value is set equal to one where M equals zero. This is the logical choice since no discounting should be applied to a failure cause that is the current cause of system failure. This also has the advantage of providing a penalty factor that will restore fractional failures to unity if a failure cause reoccurs.

To apply the Lloyd discounting method, the confidence level, γ, must be specified. The method is then applied sequentially, failure by failure, recomputing M, the number of successful trials since last failure, for each observed failure cause as the data from each trial is included. At each step, M is used in Equation 2.5 to compute a new adjusted failure value for each observed failure.

An example will help clarify the use of the Lloyd method. Applying the method to the sample data in Table 1 with γ equal to .99 yields the following results after the second failure is included.

FAILURE NUMBER	M	FAILURE CAUSE	ADJUSTED FAILURE
1	1	CAUSE A	.9900
2	0	CAUSE B	1.0000

Including the data for the third failure, due to failure Cause C, both Cause A and B are further discounted. The adjusted failure value for Cause A is computed below.

$$M = 2$$

$$\begin{aligned} \text{ADJUSTED FAILURE} &= 1 - (1 - .99)^{1/2} \\ &= 1 - \sqrt{.01} \\ &= 1 - .1 \\ &= .9 \end{aligned}$$

The complete results are shown below.

FAILURE NUMBER	M	FAILURE CAUSE	ADJUSTED FAILURE
1	2	CAUSE A	.9000
2	1	CAUSE B	.9900
3	0	CAUSE C	1.0000

The process continues sequentially, adding the next failure, recomputing M, and computing the adjusted failure values for each previous failure. The final results after all seven failures are included is shown in Table 3. Note, as a failure cause reoccurs its corresponding value for M is set to zero forcing the adjusted failure value to unity.

This concludes the description of the two discounting methods covered in the study. The following section will describe the three discrete reliability growth models.

TABLE 3

LLOYD DISCOUNTING
OF EXAMPLE DATA (TABLE 1)

TRIAL NUMBER	FAILURE NUMBER	FAILURE CAUSE A	B	C	D	ADJUSTED FAILURE
1		S	S	S	S	
2	1	F	S	S	S	1.0000
3	2	S	F	S	S	.4377
4	3	S	S	F	S	.3421
5	4	S	S	S	F	.2983
6		S	S	S	S	
7	5	S	S	F	S	.3421
8		S	S	S	S	
9		S	S	S	S	
10	6	S	F	S	S	.4377
11		S	S	S	S	
12		S	S	S	S	
13		S	S	S	S	
14		S	S	S	S	
15		S	S	S	S	
16		S	S	S	S	
17		S	S	S	S	
18	7	F	S	S	S	1.0000

S : DENOTES SUCCESS
F : DENOTES FAILURE

C. RELIABILITY GROWTH MODELS DESCRIPTION

Of particular interest in this study are test plans which terminate each test phase with system failure. Such test plans are known as testing to a fixed number of failures. Only one of the three reliability growth models proposed can be used for test scenarios

which terminate testing after a fixed number of trials. Because of this limitation, only test plans which test to a fixed number of failures are considered in this study.

Three discrete reliability growth models are presented in this section, each of which uses a different technique to track changing system reliability. The first model, Maximum Likelihood Estimate With Discounting, is derived from the conventional maximum likelihood estimate of reliability. Failure discounting gives the model the ability to track changing reliability. Two additional models derived from an exponential single phase reliability estimate referred to as the Weighted Average Model and the Exponential Regression Model are also discussed. The MLE With Discounting model will be presented in the next section.

1. Maximum Likelihood Estimate With Failure Discounting

The most commonly used estimate of system reliability is the maximum likelihood estimate Equation 2.2 restated below.

$$\underline{R} = \frac{\text{Number of Successful Trials}}{\text{Total Number of Trials}} \quad \text{(eqn 2.2)}$$

Where the number of successful trials equals the total number of trials minus the number of observed failures. Because all testing considered is conducted to a fixed

$$\text{NO. SUCCESSFUL TRIALS} = \sum_{i=1}^{k} (T_i - 1) \qquad \text{(eqn 2.6)}$$

number of failures, the number of successful trials simplifies to Equation 2.6. Where T_i equals the number of trials following the $(i-1)^{st}$ failure including the i^{th} failure and k equals the total number of failures in the test program to date.

Two problems arise when using Equation 2.2 to estimate reliability growth. First, the estimate takes a large number of trials to accurately estimate system reliability. Second, the estimator requires a constant reliability, R, for each trial. Because the reliability at each phase, R_k, may not be constant with k; only the test data from the phase of interest may be used to estimate reliability. Failure discounting is introduced into the model in an attempt to adjust the underlying reliability for data from previous phases, making it compatible with the data from the current phase. This increases the amount of data available for estimating reliability at each phase,

improving the accuracy of the estimate. Choosing the correct discounting method with the correct parameters is critical to this model because the previous test data must be properly adjusted to reflect an underlying reliability that is compatible with the current phase. This problem is difficult to solve because the actual reliability in each phase is unknown.

Introducing failure discounting creates a computation problem because the number of failures is no longer an integer value. This problem can be resolved in one of two ways. The failure can remain a fractional value or the failure value can be reset to unity by adjusting the observed number of trials up to and including failure. Which ever method is chosen, the ratio of failures to total trials must remain constant or the estimate of reliability will be altered. Returning the failure value to unity and adjusting the number of trials is the method chosen to resolve this problem. This solution allows the data to retain its original structure of the number of trials up to and including failure.

Adjusting fractional failures and the number of trials up to and including failure is accomplished by dividing both values by the fractional failure value. This returns the number of failures to unity and adjusts the number of trials such that the ratio between the number of failures and the number of trials remains constant. The adjusted number of trials may not be an integer value. This does not present a problem for computing the MLE. In cases where the total number of trials must also be an integer value, the adjusted number of trials is rounded to the nearest integer. This will have a minimal effect on the ratio of failures to trials.

For example the following data is the result of applying a failure discounting method to observed test data.

FRACTIONAL FAILURE = .5

NUMBER OF TRIALS = 6

Dividing both values by .5 adjusts the fractional failure to unity and yields an adjusted failure value.

FAILURE VALUE = 1.0

ADJUSTED NUMBER OF TRIALS = 12

The ratio of failures to trials remains constant.

$.5/6 = 1/12 = .0833$

In general, Equation 2.7 is used to adjust the number of trials when the adjusted failure

$$\text{ADJUSTED \# TRIALS} = \frac{\text{OBSERVED \# of TRIALS}}{\text{ADJUSTED FAILURE}} \qquad \text{(eqn 2.7)}$$

value is returned to unity.

The reliability growth model is applied to the data sequentially, phase by phase. After the test data from a phase is recorded, the discounting routine is applied to the data and an adjusted failure value is computed for each observed failure. The adjusted number of trials for each observed failure is computed using Equation 2.7.

$$\underline{R}_k = \frac{(\sum \text{ADJUSTED TRIALS}) - J}{\sum \text{ADJUSTED TRIALS}} \qquad \text{(eqn 2.8)}$$

J = TOTAL NUMBER OF FAILURES IN THE TEST PROGRAM TO DATE

Finally, the estimate of system reliability in phase k is computed using Equation 2.8. This procedure is repeated after each testing phase is completed yielding an estimate of system reliability for each phase. These estimates can then be used to plot the estimated reliability growth pattern.

To illustrate the use of the model, the discounted failure values from Table 2 are used to compute an estimate of system reliability. Because each data point represents a development phase, the estimate will be of phase seven reliability. The results of the computations are shown in Table 4.

Note, the sum of the adjusted trials is not rounded to an integer value. Using only the data in the seventh phase, the MLE without discounting is

$\underline{R}_7 = 7/8 = .875$.

The MLE using all test data without discounting will yield

$\underline{R}_7 = 11/18 = .611$.

For this example, the MLE with discounting provides an improved estimate of actual reliability; i.e. $\underline{R}_7 = .744$ is closer to the actual reliability value, .78.

When testing is terminated at a fixed number of failures, the MLE has an additional shortcoming; it is conservatively biased. Breaking the data into sets of trials up to and including a single failure, the number of successful trials in a set always

TABLE 4

MLE WITH DISCOUNTING
OF SAMPLE TEST DATA (TABLE 2)

PHASE #	CAUSE	ADJ. FAILURE	ADJ. TRIALS
1	A	1.0000	2.0000
2	B	.5625	1.7778
3	C	.4219	2.3702
4	D	.3164	3.1606
5	C	.4219	4.7405
6	B	.5625	5.3333
7	A	1.0000	8.0000

$k = 7$ $\qquad \sum = 27.3824$

$J = 7$

$\underline{R}_7 = (27.3824 - 7) \div 27.3824$

$= .74436$

$R = .78$

equals the number of trials in a set minus one. As such, the MLE, Equation 2.2, may be stated as follows for each set of data.

$$\underline{R} = \frac{X-1}{X}$$

X = NUMBER OF TRIALS UP TO AND INCLUDING FAILURE

The expected value of $\underline{R}$ is derived in Equation 2.9. Table 5 shows the $E[\underline{R}]$ compared to the actual reliability R. Clearly, the maximum likelihood estimate will be very conservatively biased for low reliabilities. [Ref. 3: p.34]

$$
\begin{aligned}
E[\underline{R}] &= E[\,(X-1)/X\,] \\
&= 1 - E[\,1/X\,]
\end{aligned}
$$

$$
\begin{aligned}
E[1/X] &= (1-R) + (R/2)(1-R) + (R^2/3)(1-R) + \ldots \\
&= 1 - R + (R/2) - (R^2/2) + (R^2/3) - (R^3/3) + \ldots
\end{aligned}
$$

$$
\begin{aligned}
\ln(1-R) &= -R - (R^2/2) - (R^3/3) - \ldots \\
-(1/R)\ln(1-R) &= 1 + R/2 + R^2/3 + \ldots
\end{aligned}
$$

$$
\therefore E[\,1/X\,] = \ln(1-R) - (1/R)\ln(1-R)
$$

$$
E[\underline{R}] = 1 + (1/R)\ln(1-R) - \ln(1-R) \qquad \text{(eqn 2.9)}
$$

TABLE 5

MLE EXPECTED VALUE

R	$E[\,\underline{R}\,]$
.1	.052
.3	.168
.5	.307
.7	.484
.8	.598
.9	.744
.99	.953
.999	.993

This bias can be offset by the proper choice of a discounting parameter; but, the development of a less biased estimator would be advantageous. The Exponential Single Phase Estimate, introduced in the next section, demonstrates a smaller conservative bias than the MLE when applied to geometric data.

2. Exponential Single Phase Reliability Estimate

The Exponential Single Phase Reliability Estimate was developed for the special case of testing until the first failure is encountered. Test data of this form has a geometric distribution with the parameter, R, system reliability. As shown in the previous section, the maximum likelihood estimate is conservatively biased when applied to geometric data. In an attempt to reduce this bias, an estimator of the following form was suggested.

$$R = 1 - e^{-A}$$
$$e^{-A} = 1 - R$$
$$A = -\ln(1 - R)$$

To complete the development of the estimator, an unbiased estimate, $\underset{\sim}{A}$, for A was sought such that

$$E[\underset{\sim}{A}] = -\ln(1 - R) = A$$

The estimate is derived as follows. Let X_i equal the number of trials up to and including failure for the i th set of geometric data and R_i equal the system reliability for the i th set of data where i = 1, 2, 3, ... , number of failures. Assume that $Y_i = f(X_i)$ is an unbiased estimate of A. Then

$$\sum_{x_i=1}^{\infty} f(x_i)\ R_i^{x_i - 1}\ (1 - R_i) = -\ln(1 - R_i)$$
$$= R_i + R_i^2 + R_i^3 + \dots .$$

Solving for $f(x_i)$ by equating coefficients of terms with like powers yields

$$f(x_i) = 1 + 1/2 + 1/3 + \dots + 1/(X_i - 1).$$

Resulting in Equation 2.10.

$$Y_i = 1 + 1/2 + \dots + 1/(X_i - 1) \qquad \text{for } X_i \geq 2 \qquad \text{(eqn 2.10)}$$
$$= 0 \qquad \text{for } X_i = 1$$

Substituting Y_i, an unbiased estimate of A, into the reliability equation yields an estimate of single phase reliability, Equation 2.11. Equation 2.11 can be used to estimate system reliability when testing is performed until first failure.

As has been shown, Y_i is an unbiased estimate of A; but, this does not ensure Equation 2.11 will provide an unbiased estimate of system reliability. To resolve this

$$\underline{R} = 1 - e^{-Y_i} \qquad \text{(eqn 2.11)}$$

question Monte-Carlo simulation was used to evaluate the properties of the Exponential Single Phase Estimate and compare it with the MLE.

The algorithm described below was used to evaluate the Exponential Single Phase Estimate, Equation 2.11. First a uniform (0,1) random number was generated. A table look-up algorithm was used to convert the uniform (0,1) random variable to a geometric random variable with reliability R. Estimates of system reliability, based on the geometric random variable, were computed using both the Exponential Single Phase Estimate and the Maximum Likelihood Estimate. The procedure was then replicated a large number of times. The running average of the estimates for each estimator was updated after each replication. The results of the simulation are shown in Table 6 for ten-thousand replications.

TABLE 6

ESTIMATE EXPECTED VALUE
FOR MLE & EXPONENTIAL ESTIMATE

THEORETICAL RESULTS		SIMULATION RESULTS	
R	E[MLE]	MEAN MLE	MEAN EXP. EST.
.1	.052	.053	.066
.2	.107	.107	.132
.3	.168	.170	.207
.4	.234	.234	.281
.5	.307	.305	.361
.6	.389	.388	.452
.7	.484	.484	.552
.8	.598	.598	.664
.9	.744	.744	.798
.99	.953	.954	.968
.999	.993	.994	.996

The simulation generated mean for the MLE is very close to its theoretically derived value verifying the accuracy of the simulation. Comparing the simulation generated mean estimate for the Exponential Single Phase Estimator to the actual reliability leads to the conclusion that the estimate is conservatively biased, but less biased then the MLE. As is the case with the MLE, the amount of bias in the Exponential Single Phase Estimate is a function of the actual reliability.

Constant system reliability is necessary for the Exponential Single Phase Estimate to accurately predict reliability. Because of this property, the estimator must be modified to accurately predict the changing system reliability inherent in a reliability growth pattern. As was the case with the MLE, the modification should make it possible to use all available test data when computing a reliability estimate. The remaining reliability growth models explore alternative modifications to the Exponential Single Phase Estimate. The first modification employs linear regression to estimate the parameter A. The second modification involves taking the weighted average of the current single phase estimate and the preceding phase reliability estimate to produce an estimate of current phase reliability.

3. Exponential Regression Discrete Reliability Growth Model

The Exponential Regression Reliability Growth Model was developed by W. M. Woods from the Exponential Single Phase Reliability Model [Ref. 4]. The model studied here is identical to the original model with the exception of applying failure discounting. The model can only be applied in situations where testing is performed to a fixed number of failures and must have attribute data. Any failure discounting method may be used with the model; but, the discounted failure data must be returned to its original form, an integer number of failures and successes. To apply the model, one must record the data necessary to apply the chosen failure discounting method, the total number of failures and successes, and the number of successful trials between each failure. The equation

$$R = 1 - e^{-A}$$

where

$$A = \alpha + \beta k$$

and k denotes the testing phase is used to compute system reliability

$$R = 1 - e^{-(\alpha + \beta k)}$$

$$k = 1, 2, 3, \ldots$$

Estimating the expression $\alpha + \beta k$ at each phase allows the model to track changing reliability. The model is further modified to include the possibility of more than one failure in a given phase. Let F_k equal the total number of failures in the k th phase and j equals the failure number in phase k such that $j = 1, 2, ..., F_k$. N_{jk} equals the number of trials between the $(j - 1)^{st}$ and the j th failure, including the j th failure, in phase k. As in the Exponential Single Phase Estimate, Equation 2.10, still represents an unbiased estimate of $(\alpha + \beta k)$ for each set of trials N_{jk}. To avoid confusion with the additional subscript, Equation 2.10 becomes Equation 2.12

$$\begin{aligned} Y_{jk} &= 1 + 1/2 + ... + 1/(X_i - 1) \qquad && \text{for } X_i \geq 2 \qquad \text{(eqn 2.12)} \\ &= 0 && \text{for } X_i = 1 \end{aligned}$$

Estimates of α and β for each phase are developed using the techniques of linear

$$\underline{\beta}_k = \frac{\sum_{j=1}^{k} (j - K.) \times Y_{\cdot j}}{\sum_{j=1}^{k} (j - K.)^2} \qquad \text{(eqn 2.13)}$$

regression yielding Equations 2.13 and 2.16. Equation 2.13 requires the computation of $Y_{\cdot j}$, Equation 2.14, and K., Equation 2.15.

$$Y_{\cdot k} = (Y_{1k} + Y_{2k} + ... + Y_{F_k\,k})/F_k \qquad \text{(eqn 2.14)}$$

$$K. = (1 + 2 + ... + k)/k \qquad \text{(eqn 2.15)}$$

$$\underline{\alpha}_k = Y.. - \beta_k K. \qquad \text{(eqn 2.16)}$$

$$Y_{\cdot\cdot} = (Y_{\cdot 1} + Y_{\cdot 2} + \ldots + Y_{\cdot k})/k \qquad \text{(eqn 2.17)}$$

Substituting the estimates for α_k and β_k into the reliability equation yields the estimate of reliability for phase k, Equation 2.18.

$$\begin{aligned} \underline{R}_k &= 1 - e^{-(\alpha_k + \beta_k k)} & k > 1 \qquad \text{(eqn 2.18)} \\ &= 1 - e^{-Y_{\cdot 1}} & k = 1 \end{aligned}$$

A separate definition of the case where $k = 1$ is necessary because regression techniques require a minimum of two data points. [Ref. 3: pp.1-4]

The only modification made to the original model is the addition of failure discounting. After each phase, the chosen failure discounting method is applied to the test data. The fractional failures that result must be adjusted to unity by adjusting the number of trials, Equation 2.7. Computation of Y_{jk} requires the total number of trials be returned to an integer value. Therefore, the adjusted trials are rounded to the nearest integer. Estimates for β, Equation 2.13, and α, Equation 2.16, are then computed. These values are substituted into Equation 2.18 to obtain the estimate of system reliability for phase k.

Because the parameters α and β are computed using linear regression, the model has the ability to track changing reliability even if no failure discounting is applied. Even so, it may take numerous phases and a great deal of data for the Exponential Regression Model to accurately predict reliability. Adding failure discounting to the model should decrease the amount of data required for an accurate estimate of system reliability.

The Exponential Regression Model can predict a negative system reliability. This occurs most often in the initial testing phases and at low reliabilities. Should this occur, the estimate should be set to zero.

An example will help clarify the application of the model. Using the discounted test data from Table 2, the estimate of phase seven reliability is computed in Table 7.

Note the Y_{jk} column was deleted because each phase consists of a single failure and $Y_{jk} = Y_{\cdot k}$. This example also demonstrates the results of applying improper discounting parameters for the situation. Failure discounting caused the estimator to further overestimate the actual reliability. Had no discounting been applied, the regression estimate would have yielded $\underline{R}_7 = .836$.

TABLE 7

EXPONENTIAL REGRESSION ESTIMATE COMPUTATIONS

k	ADJ. TRIALS	$Y_{\cdot k}$	$(k - K.)Y_{\cdot k}$	$(k - K.)^2$
1	2	1.0	−3.0	9
2	2	1.0	−2.0	4
3	2	1.0	−1.0	1
4	3	1.5	0.0	0
5	5	2.083	2.083	1
6	5	2.083	4.166	4
7	8	2.593	7.779	9
	$\sum$	11.259	8.023	28

$K. = 4.0$

$Y.. = 11.259/7 = 1.608$

$\beta_7 = (8.028 \div 28) = .287$

$\alpha_7 = 1.608 - (.287 \times 4) = .460$

$(\alpha_7 + \beta_7 K) = .460 + (.287 \times 7) = 2.469$

$\underline{R}_7 = 1 - \exp(-2.469) = .915$

$R_7 = .78$

The last remaining reliability growth model is also derived from the Exponential Single Phase Estimate. A weighted average technique is used to allow the estimator to track changing reliability. This estimator is described in the next section.

4. Weighted Average Discrete Reliability Growth Model

The Weighted Average Model tracks changing reliability by taking a weighted average of the two most recent reliability estimates. This model is only applicable to situations when testing is performed until a fixed number of failures have been observed and attribute data is available. A slightly modified version of the Exponential Single Phase Reliability Estimate is used in the model. The current phase reliability is estimated using a weighted average of the previous phase reliability estimate and the

current single phase estimate. The previous phase estimate and the current single phase estimate are weighted by the fraction of total trials to date used in the computation of each estimate. As with the Exponential Regression Estimate, this model will track changing reliability without the use of failure discounting; but, like most weighted averages, the model may lag actual growth by one or more phases. Failure discounting is used to correct the lag and reduce the number of trials necessary to compute an accurate estimate.

Equation 2.11 is applicable when testing to first failure. A slightly different version of the estimator is applicable to situations where more than one failure has been recorded. To apply this estimator, a running total of the number of successful trials to date, S_k , and failures to date, F_k, is maintained for each phase k. These totals are used in Equation 2.19 to compute the current single phase reliability estimate, $\underline{R}_{sp_k}$.

$$\underline{R}_{sp_k} = 1 - e^{-\underline{C}_k} \qquad \text{(eqn 2.19)}$$
$$\underline{C}_k = 1/F_k + 1/(F_k + 1) + \ldots + 1/[(F_k + S_k) - 1]$$

For this model, running totals of failure and success data are recorded for each observed failure cause. As each trial is performed, the number of successes for each failure cause, except the current failure cause, is incremented by one. Should a previously recorded failure cause reoccur, its failure total is incremented by one. The system values for F_k and S_k are then selected from the individual failure cause having the largest number of failures to date.

Once the single phase estimate is computed, the estimate of system reliability is computed by taking the weighted average of the previous phase reliability estimate

$$\underline{R}_k = \frac{N_1}{N}\underline{R}_{k-1} + \frac{N_2}{N}\underline{R}_{sp_k} \qquad k=2,3,\cdots \qquad \text{(eqn 2.20)}$$
$$= \underline{R}_{sp_k} \qquad k=1$$

and the current single phase estimate, Equation 2.20.

Where N equals the total number of trials recorded to date. N_1 equals the total number of trials recorded in previous phases. N_2 equals the number of trials recorded in phase k. Note the sum of N_1 and N_2 equals N.

Failure discounting is applied to the model in much the same way as it was in the Exponential Regression Model. After a discounting method has been applied to the data, fractional failures are returned to unity by adjusting the number of trials, Equation 2.7. The adjusted number of trials is then rounded to the nearest integer. It is important to note, when failure discounting is applied to this model S_k must be recomputed from phase one at each phase boundary because the recorded number of successful trials for each previous phase may be altered by the discounting routine.

An example of the model's application should help clarify the required computations. Using the discounted failure data from the example of the Straight Percent Discounting Method leading up to Table 2, at the end of phase one the sum of failures and successes for the failure cause is listed below.

	FAILURES	SUCCESSES
CAUSE A	1	1

Solving for the single phase estimate and the reliability estimate yield

$$\underline{C}_1 = 1.0$$
$$\underline{R}_1 = \underline{R}_{sp_1} = 1 - e^{-1} = .6321 \text{ .}$$

After the second phase data is included and the failure discounting method applied, the sum of failures and successes for each failure cause is listed below.

	FAILURES	SUCCESSES
CAUSE A	1	2
CAUSE B	1	2

From the failure data, F_2 equals one and S_2 equals two yielding $\underline{C}_2$ equal to 1.5. Applying the remaining model yields the second phase reliability estimate $\underline{R}_2$.

$$\underline{R}_{sp_2} = 1 - e^{-(3/2)} = .7769$$
$$N_1 = 2$$
$$N_2 = 1$$
$$N = 3$$
$$\begin{aligned}\underline{R}_2 &= (2/3)\,\underline{R}_1 + (1/3)\,\underline{R}_{sp_2} \\ &= (2/3)(.6321) + (1/3)(.7769) \\ &= .6804\end{aligned}$$

The computations are repeated for each phase of the testing program. As mentioned previously, when failure discounting alters the number of successful trials, S_k, in a previous phase, the reliability estimates for all previous phases must be recomputed to solve for the current phase reliability estimate. As this simple example clearly shows, this reliability growth model is computationally intense. Programming the model on a computer may be necessary to compute reliability estimates for large data sets.

Two failure discounting methods and three discrete reliability growth models have been described in this chapter. Evaluation of each model is very difficult because the expected value and variance of the estimates, when discounting is applied, become mathematically untractable. It is also not possible to evaluate the models using actual test data because the true reliability at each phase is unknown. The following chapter discusses a Monte-Carlo simulation written to evaluate the performance of each of the failure discounting methods and discrete reliability growth models.

III. DISCRETE RELIABILITY GROWTH SIMULATION

Evaluating reliability growth estimators including reliability growth models, is often difficult. True system reliability is unknown, rendering actual test data useless for evaluating an estimators performance. When the mathematics is tractable, the properties of the estimator can be derived and used to evaluate the estimators performance. However, many estimators are too complex to allow mathematical solutions for the closed form of their properties. For this group of estimators, computer simulation is one of the only evaluation techniques available. The addition of failure discounting to an estimator makes mathematical solutions very difficult if not impossible. The reliability growth models introduced in this study fall in the category requiring simulation as an evaluation tool.

A Monte-Carlo simulation was written to evaluate the properties of the three reliability growth models introduced in Chapter II. The simulation builds a reliability growth pattern and generates test data from the known system reliabilies using the geometric distribution. Each of the reliability growth models are then applied to the simulation generated test data. Repeating the process many times for a particular set of known reliabilities will provide a good estimate of both the mean and variance of each estimator as well as provide information about the underlying distribution of the estimate. These statistics can then be used to evaluate each reliability growth model's performance.

Several modeling problems had to be solved before the simulation could be built. A method for generating a realistic reliability growth pattern had to be developed. This method must closely model the process of testing equipment, implementing design improvements or "fixes" and continued testing. It also must providing the flexibility to test numerous reliability growth patterns. Once the actual system reliability in each phase is established from the growth pattern, test data of the form the number of trials up to and including failure must be generated. Because such data are geometrically distributed when R is fixed, this process is relatively simple. A relistic method must also be developed for determining the cause of each failure. This step proved to be the most difficult.

As with all simulations, several modeling assumptions were necessary. The following section discusses the assumptions and their impact on the simulation. The remainder of the chapter describes the development of the simulation algorithm and its capabilities and limitations.

A. MODELING ASSUMPTIONS

Building an algorithm that will generate realistic test data requires several simplifying assumptions about the actual test process. These assumptions can be broken into three categories. They are:

1. General assumptions
2. Assumptions concerning the true reliability growth pattern
3. Assumptions concerning failure cause determination

The last two categories correspond to the two steps in generating test data, generating the actual reliability growth pattern and computing test data from the growth pattern. Each assumption is listed and discussed by category.

1. General Assumptions

The general assumptions narrow the scope of testing considered to those scenarios which apply to the reliability growth models evaluated. The two required assumptions are:

1. All testing is performed to a fixed number of failures.
2. All test results are recorded as attribute data.

These assumptions ensure the test data produced by the simulation will be compatible with the discrete reliability growth models developed in the study. The models can only be applied to discrete or attribute data where testing is performed until a fixed number of failures are observed.

2. Reliability Growth Pattern Assumptions

Defining the mechanisms which cause an items' reliability to change from phase to phase is necessary before an algorithm to generate a realistic reliability growth pattern can be written. Several assumptions were necessary to properly define these mechanisms. They are:

1. The reliability growth pattern is non-decreasing.
2. System reliability changes only at phase boundaries.
3. Equipment improvements are implemented immediately after a phase ends and before any further testing.

4. Only design weaknesses causing failure during a phase are corrected at the end of a phase.
5. Each design improvement removes a fixed fraction of the probability of reoccurrence for the corresponding failure cause.

The first assumption limits the shape of the generated growth patterns to non-decreasing curves. Because one of the objectives of development testing is to improve system reliability, this assumption is logical. However, actual reliability growth may have periods of decline when "fixes" to design problems actually decrease reliability. Over the entire testing period, even these curves are generally increasing. The worst case this assumption allows is no growth over the testing phases. This should be adequate for determining adverse growth characteristics of the reliability growth models.

Assuming system reliability remains constant throughout a testing phase makes the model consistent with the previous definition of a testing phase. This assumption means no design changes or any other improvements are made to an item on test within a phase. Because a phase can be defined as any number of failures greater than or equal to one, this assumption should have little impact upon the accuracy of the simulation.

Assuming all fixes are implemented after a test phase ends and before the next phase begins is closely related to the previous assumption. This assumption does preclude accurate modeling of long term fixes whose implementation are postponed while testing continues. The impact of this limitation should be minimal.

Assumption four asserts that the only known weaknesses in system design are those which have caused system failure. This is not always the case. Suspected weaknesses may be corrected along with those weaknesses confirmed by system failure. The simulation is unable to model this situation. Even though this deviates somewhat from reality, most major improvements to system reliability should come from failure identified weaknesses. The intent of the simulation is only to generate a realistic reliability growth pattern not to duplicate an actual growth pattern. Therefore the simulation results should be adequate to accomplish the task for which the model was designed.

The last assumption in this category deals with the mechanism used by the simulation to model improvements in system design. It asserts that all fixes remove a constant fraction of the probability of occurrence for the failure cause of interest. In reality many factors determine how effective a particular fix will be and all fixes are not

equally effective. This method also decreases the probability of occurrence more for a failure cause with a high probability of occurrence than for a failure cause with a low probability of occurrence. This also is unrealistic. Despite the shortcomings of this simplification, it does provide a simple way to model reliability improvement due to design change and will produce a growth pattern that is adequate for evaluating the reliability growth models studied.

The final category of assumptions deal with generating test data from a known system reliability and determining the cause of each failure. They are discussed in the next section.

3. Failure Cause Determination Assumptions

Realistically modeling failure causes is critical to the evaluation of the failure discounting methods. The technique used to model failure cause determination must also be incorporated into the technique used to model reliability growth because the two processes are closely related. The assumptions made in modeling failure cause determination are:

1. There exists a finite number of possible failure causes.
2. Each failure cause has a fixed probability of occurrence in each phase.
3. System reliability can be modeled as a series system of the failure causes.
4. Each failure cause is stochastically independent of the other failure causes.

Assuming a fixed number of failure causes may be an oversimplification of reality but should not invalidate the model. Even though the possible number of failure causes is most likely infinite, only a finite number of these causes with relatively high probabilities of occurrence are of interest. The number of possible failure causes considered by the simulation must be specified as an input to the model; but, the number of causes input is only limited by the capacity of the computer on which the model is run. Therefore, the simulation is flexible enough to model nearly any scenario of interest.

Fixing the probability of occurrence for all failures causes within a single phase is a direct result of fixing the system reliability for all trials in a single phase. This assumption will not further restrict the ability of the algorithm to model reality.

An assumption is needed to interrelate failure cause determination and system reliability growth through design improvements. Assuming the component being tested may be modeled as a series system of failure causes meets this requirement. System reliability at any phase is a function of the probability of occurrence for the specified

failure causes. As such, implementing a design improvement reduces the probability of occurrence for a failure cause which in turn increases the systems reliability. This assumption also makes it possible to stochastically determine the cause of failure. The mathematical derivation of this technique will be presented in a later section. Modeling the failure cause as a series system does not depart far from reality because the occurrence of a failure cause implies system failure. Once the assumption of a finite set of failure causes is accepted, modeling them as a series system does not further restrict the models representation of reality.

The final assumption requires that each failure cause be stochastically independent. This means that the causes of failure are not related. The occurrence of one cause does does not influence the probability of occurrence for any other cause. If the failure causes are properly defined, this assumption should not limit the accuracy of the model.

The following section discusses the simulation algorithm built from these assumptions.

B. SIMULATION ALGORITHM DEVELOPMENT

The algorithm developed for this simulation can be broken into the four sections. They are:

1. Generate a reliability growth pattern.
2. Generate test data, including the cause of failure, from the reliability growth pattern.
3. Apply the failure discounting methods and reliability growth models to the simulation generated data.
4. Compute statistics used to evaluate the performance of each model.

The first two steps of the algorithm along with the output statistics will be discussed in detail in this section. The methodology used to apply each of the discounting methods and reliability growth models was discussed in Chapter II and will receive little attention here. With few exceptions, the actual content of the Fortran code used in the simulation will not be discussed. A documented copy of the code is included at Appendix A for the interested reader.

1. Simulation of a Reliability Growth Pattern

The reliability growth pattern built by the simulation models the actual process of testing and improving an item in a development program. Several user inputs are required at this stage of the simulation. They are:

1. number of testing phases

2. number of failures in each testing phase
3. number of possible failure causes
4. the first phase probability of occurrence for each possible failure cause
5. growth fraction (F_g)

Once these parameters are in place, the simulation computes the first phase system reliability from the probability of occurrence for each possible failure cause. The number of trials up to and including failure and the failure cause are then computed for all failures in the first phase. The algorithm used to generate system reliability and the failure data will be covered in the next section.

After the failure data for the first phase has been generated, the probability of occurrence for each failure cause which occurred in the first phase is recomputed using the growth fraction. Assume the first failure occurred due to cause A, the new probability of occurrence, Pr $(\text{occurrence})_n$, will be computed using Equation 3.1.

$$\Pr(\text{occurrence})_n = \Pr(\text{occurrence}) \times (1 - F_g) \qquad \text{(eqn 3.1)}$$

This equation is applied at the end of each phase to those failure causes that occurred in the current phase. If a failure cause is responsible for multiple failures in the phase, Equation 3.1 is applied only one time. This procedure simulates identifying a system weakness through testing and implementing a design improvement. The procedure is repeated at the end of each testing phase.

The result of the algorithm is a stochastically determined reliability growth pattern that simulates actual developmental testing. A test is performed, weaknesses in the system are identified, corrective design changes are implemented and testing is continued. This algorithm also has the advantage of interrelating the probabilities of occurrence for each failure cause and the system reliability. This will be useful in determining the failure cause. By changing the growth fraction, the rate of reliability growth can be varied from no growth to rapid growth.

2. Generating Test Data From Known Reliability

Two problems must be solved before an algorithm that generates test data can be developed. The system reliability must be determined from the probability of occurrence for each failure cause. A technique for generating the number of trials up to and including failure from known reliability that identifies the failure cause also must be developed. The solution to both problems lies with the assumption that system

reliability can be modeled as a series system of the failure causes. In a discrete series system the number of trials up to and including system failure will be the minimum of the number of trials up to and including failure for each sub-component. Let M equals the number of trials up to and including failure for the system and X_i where $i = 1, 2, \ldots, j$ denotes the number of trials up to and including failure for the i th subcomponent, here a failure cause. If the j components form a series system, M may be expressed as

$$M = MIN\,(X_1, X_2, \ldots, X_j)$$

Because the probability of occurrence is fixed throughout a phase for each failure cause, X_i is distributed as a geometric random variable with parameter R_i. From an assumption, the X_i's are also mutually independent. It can be shown for a geometric random variable, X_i, that

$$Pr\,(X_i > n) = R_i^n.$$

The distribution for M is derived as follows.

$$\begin{aligned} M &= MIN\,[\,X_1, X_2, \ldots, X_j\,] \\ Pr\,[M > n] &= Pr\,[\,X_1 > n, X_2 > n, \ldots, X_j > n\,] \\ &= Pr\,[X_1 > n] \times Pr\,[X_2 > n] \times \ldots \times Pr\,[X_j > n] \\ &= R_1^n \times R_2^n \times \ldots \times R_j^n \\ &= [\,R_1 \times R_2 \times \ldots \times R_j\,]^n \end{aligned}$$

Therefore M is also distributed as a geometric random variable with the parameter

$$R = (\,R_1 \times R_2 \times \ldots \times R_j\,).$$

The system reliability at each phase is computed as the product of the compliments of the probabilities of occurrence for each possible failure cause. Because of this computation the actual simulation input is the compliment of the first phase probability of occurrence for each failure cause. As the probability of occurrence for each failure cause is recomputed from phase to phase, the system reliability is also adjusted.

The number of trials up to and including failure for a geometrically distributed random variable with reliability R can be generated from a uniform (0,1) random variable, U, using Equation 3.2. [Ref. 5].

$$X = INT\,[\,1 + (\,\ln(U)/\ln(R)\,)\,] \qquad \text{(eqn 3.2)}$$

Equation 3.2 combined with the definition of a series system provide the solution to generating the number of trials up to and including failure while accounting for the cause of failure.

For each failure generated by the simulation, Equation 3.2 is used to generate a geometric random variable for each failure cause using the compliment of the probability of occurrence as the parameter. The minimum geometric random variable over the failure causes becomes the number of trials up to and including failure for the system. The cause of system failure is recorded as the failure cause producing the minimum geometric random variable. Generating the test data in this fashion allows the reliability growth pattern and test data to be stochastically generated yet interrelated much as they are in nature. A summary of the steps required to compute the reliability growth pattern and the test data follows.

1. Input the required data
2. Generate phase failure data including the number of trials and the failure cause.
3. Recompute the probability of occurrence for the failure causes that occur in the phase.
4. Return to step 2 until all phases are complete.

Once the test data has been computed from the known reliability in each phase, the failure discounting methods and reliability growth models can be applied to the test data. The resulting reliability estimates can then be evaluated against the known system reliability.

3. Application of the Discounting Methods and Reliability Growth Models

The discounting methods and reliability growth models are applied to the generated test data in much the same way as they were described in Chapter II. Each of the three reliability growth models are applied to each set of test results. However, the simulation user must specify one of the two failure discounting methods because the simulation can not apply both simultaneously. The maximum likelihood estimate without discounting is also computed for each phase, independent of all other test data. This allows a comparison between the standard reliability estimator and the reliability growth models. The simulation produces an estimate of system reliability from each reliability growth model for each testing phase. The single phase MLE and actual system reliability are provided as a basis for comparison.

One minor departure from the application of the reliability growth models as described in Chapter II was necessary. For large numbers of trials up to and including failure, N, computations of the form

$$Y = 1 + (1/2) + \ldots + [1/(N-1)]$$

required extensive computer processing time to solve recursively. To improve program efficiency, the values for Y up to and including one-thousand were computed exactly and stored in an array. For values of N greater than one-thousand non-linear regression was used to estimate the value of Y. Equation 3.3 provided an adequate estimate with an R^2 value of .99635.

$$Y = 3.55445 \times N^{(11/100)} \qquad \text{(eqn 3.3)}$$

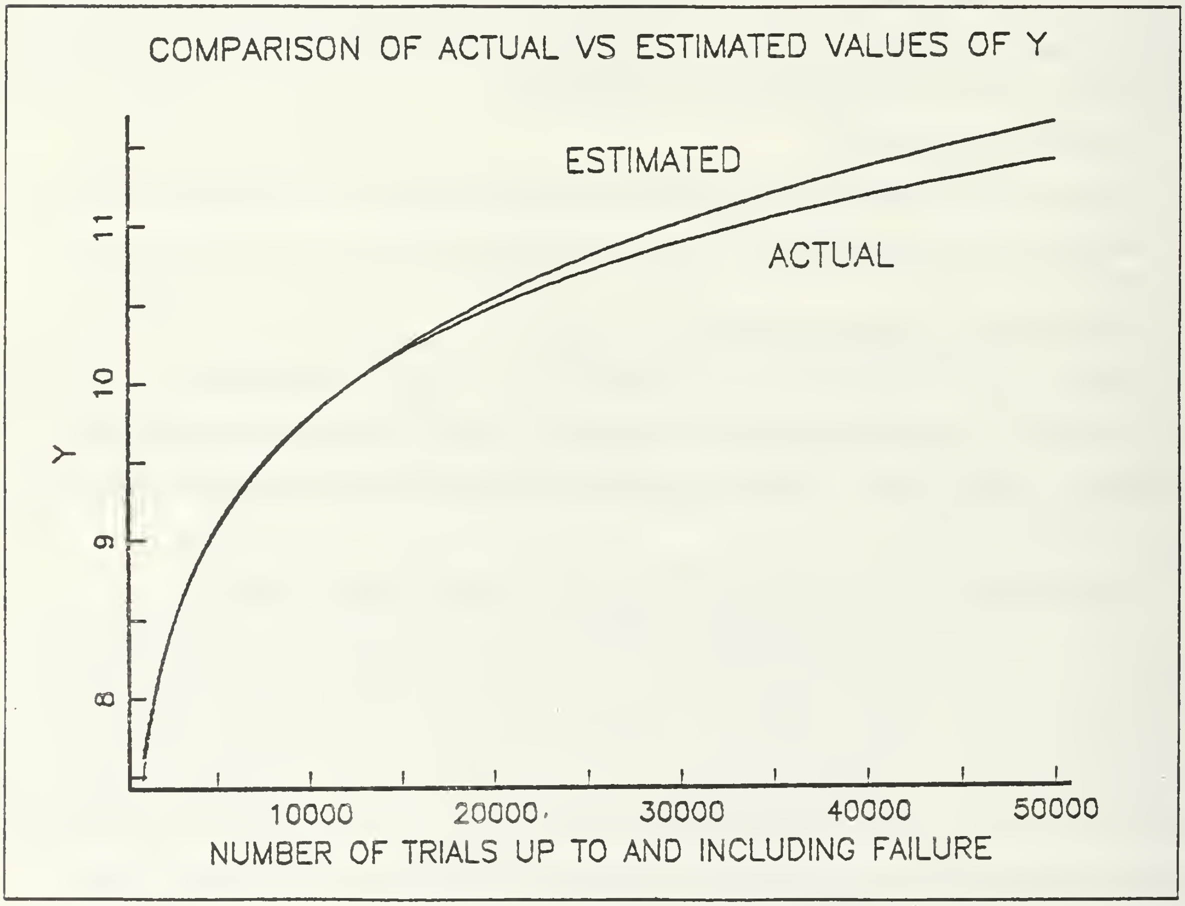


Figure 3.1 Regression Estimate of Y.

Figure 3.1 compares the actual Y value with the regression predicted value for large N. Both the Weighted Average Model and the Exponential Regression Model are affected by the approximation. The approximation has minimal impact upon the computation of model estimates. Values for N above one-thousand are rare even at high reliability

and values above 15,000, the accurate extension of the approximation, are very rare. Should a value of N larger than 15,000 occur, some error will be introduced into the model estimate, and this error will increase with N. Because these values are rare, this error is not considered significant.

Further research, after the simulation was written, has yielded a more accurate

$$Y = C + \log_e(N) + 1/(2N) - 1/[12N(N+1)] - 1/[12N(N+1)(N+2)] \qquad \text{(eqn 3.4)}$$
$$C = \text{Euler's Number} = 0.5772156648$$

approximation for Y. This series solution is shown in Equation 3.4 [Ref. 6]. This equation should be incorporated into the simulation to improve accuracy in future runs.

The estimates resulting from the application of the reliability growth models are retained through the desired number of replications to allow the computation of descriptive statistics. These statistics aid in evaluating each growth model.

4. Statistical Summary of Model Results

Several descriptive statistics are generated by the model that aid in evaluating the performance of each reliability growth model. They include the mean and standard deviation of the reliability estimates at each phase for each of the growth models. A confidence interval is also computed for the mean estimate at each phase.

Replications of the simulation are required to compute the descriptive statistics. After the first replication, the actual system reliability in each phase is fixed. At each replication, new test data is generated from the fixed reliability growth pattern and each of the reliability growth models is applied to the data. The estimates generated by the growth models are retained at each replication. When the desired number of replications have been completed, the sample mean and sample standard deviation for each estimator are computed using standard formulas. A 95% confidence interval for the mean estimate is computed for each phase reliability estimate with the assumption that each estimate is normally distributed. The assumption of normality is not valid in every case but the confidence interval is included as a tool for comparison. Further analysis of the reliability growth models can be accomplished by saving each estimate in an output file. The models user can then analyze the data as desired.

Changing the simulations' input parameters makes it possible to analyse a wide variety of testing situations. These capabilities are discussed in the following section.

C. CAPABILITIES OF THE MODEL

Within the limits of the assumptions stated earlier, the simulation is very flexible. Any number of failure causes may be input into the model. By varying the phase one probability of success for each failure cause, the growth fraction and the number of phases a wide range of reliability growth curves may be explored. The random number generator seed is also user specified allowing the exact string of random numbers and test data to be duplicated. In this way, various discounting methods and parameters may be evaluated using the exact same data.

The test-fix-test scenario may be modeled by specifying a single failure per phase. The test-find-test methodology may be simulated by specifying multiple failures in each phase. There is no requirement that each phase have an equal number of failures. Therefore a mixed scenario, test-fix-test and test-find-test, may be simulated.

Each of the discounting methods described in Chapter II may be applied to the data using the full range of input parameters. Proper parameter selection for the discounting method will cause no discounting to occur. In this manner each of the reliability growth models may be evaluated without failure discounting applied.

The simulation provides a tool for evaluating the reliability growth models against a known system reliability growth pattern. When large numbers of replications are run, information concerning the expected value and variance of each estimator may be obtained. The combinations of failure discounting parameters, reliability growth models and anticipated growth patterns may also be evaluated. The following chapter discusses the results of numerous runs of the simulation with varying growth patterns and failure discounting parameters.

IV. EVALUATION OF RELIABILITY GROWTH MODELS

The discounting methods and reliability growth models were evaluated using the Monte-Carlo simulation described in Chapter III. The results from numerous runs of the simulation with varying reliability growth patterns, failure discounting methods and failure discounting parameters were reduced to graphical output for evaluation.

The purpose of the evaluation was to determine general characteristics of each reliability growth model. The number of simulation runs was not adequate to prove that conclusions reached here will hold for all cases. Also, no fast rules for model selection or failure discounting parameter selection were developed. Still, many characteristics of the different reliability growth models were apparent from the simulation runs completed. In particular the answers to the following questions were sought.

1. How well does the reliability growth model track the actual reliability growth pattern?
2. How did the choice of failure discounting parameters affect the estimates of reliability?
3. How variable were the estimates provided by each of the models?

It was impossible to evaluate all possible reliability growth patterns that can be produced by the simulation. Five widely different reliability growth patterns were chosen to be representative of the possible range. Constant reliability cases were included to identify models that predict reliability growth, were in fact, none exists. The five reliability growth patterns chosen are described below.

1. low reliability with no growth
2. high reliability with no growth
3. low initial reliability with moderate growth
4. moderate initial reliability with moderate growth
5. low initial reliability with rapid growth

All simulation input parameters were kept fixed to insure the different simulation runs could be compared. The number of phases was held constant at ten with one failure per phase. Five failure causes were used and each simulation was replicated one-hundred times. The seed for the random number generator was kept constant for all simulation runs insuring the same string of random numbers and, therefore, identical test data was generated for each growth pattern.

Each reliability growth model is evaluated separately in the following sections. The graphical results of the remaining simulation runs are included in Appendix B.

A. GENERAL RESULTS

Characteristics common to all the reliability growth models are stated here for convenience.

The variability of each of the estimators decreased with increasing phases. This is due to the increased data available to the model with each additional phase.

The accuracy of each estimate increased and the variability of each estimate decreased with increasing reliability. This is a function of testing until a fixed number of failures have been observed. As reliability increases, the average number of trials required to produce failure will also increase. Therefore, as reliability increases, more data is available for the model improving the accuracy of the estimate and decreasing the estimate's variability.

B. WEIGHTED AVERAGE RELIABILITY GROWTH MODEL

The Weighted Average Reliability Growth Model proved to be ineffective as an estimator of changing reliability. The model consistently overestimated the actual reliability even when no failure discounting was applied. Figure 4.1 plots the mean estimate from each model at each phase. Clearly the mean phase estimates for the Weighted Average Model overestimated the actual reliability at each phase. The addition of failure discounting only amplified the tendency to overestimate as seen in Figure 4.2. Even though the variance of the estimates decreased rapidly with increasing phases, the model is not useful in its current form. Therefore, the Weighted Average Model was not considered in any further analysis.

C. MAXIMUM LIKELIHOOD ESTIMATE WITH FAILURE DISCOUNTING

The MLE With Discounting Model accurately tracked a wide range of reliability growth patterns. With proper selection of the failure discounting method and parameter, this model tracked each of the reliability growth patterns tested. Figures 4.2, 4.3, 4.4, 4.5, and 4.6 display the model's performance for each of the five reliability growth patterns tested. The model is very sensitive to the selection of the failure discounting method and parameter. In general, the model underestimates actual reliability in the early phases and slowly converges to the actual value with increased test data.

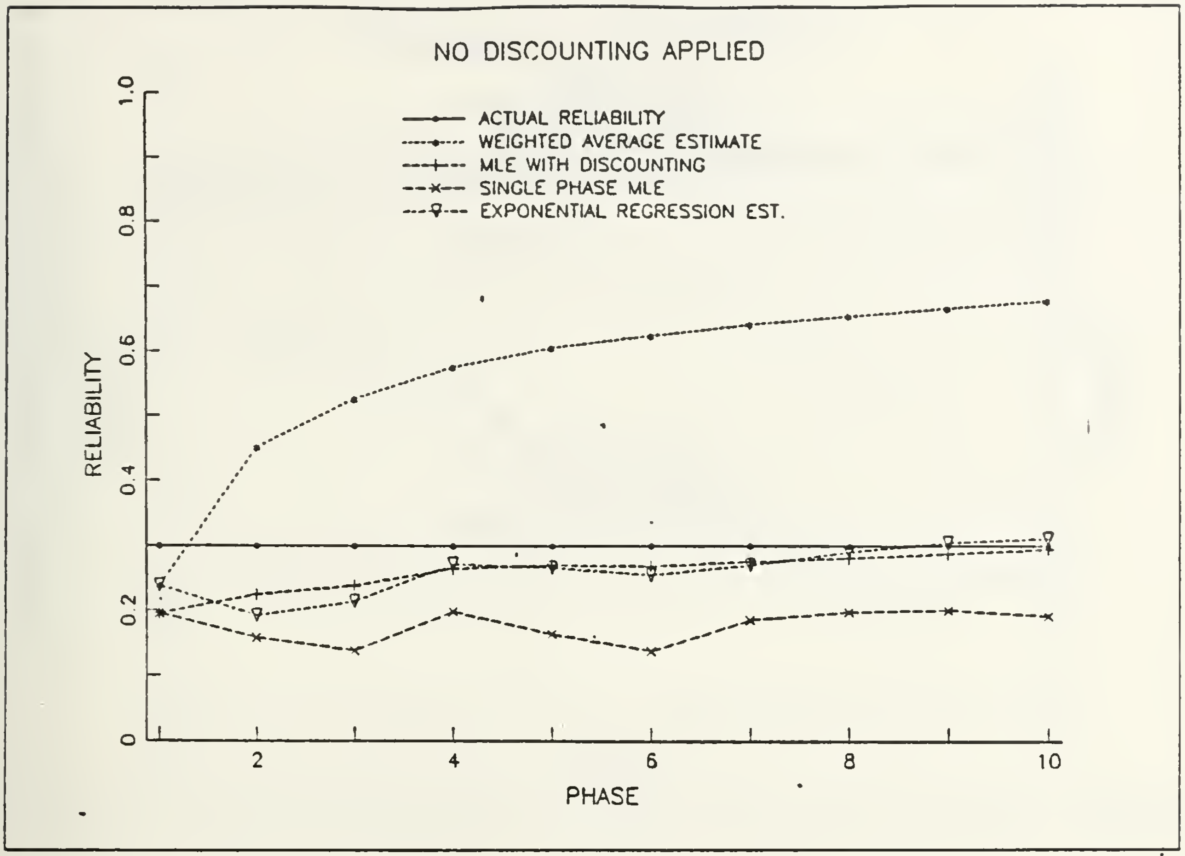


Figure 4.1 Reliability Growth Models
Low Constant Reliability
No Discounting Applied.

The choice of failure discounting method and parameter is critical to the MLE With Discounting Model. Because the Lloyd Discounting Method does not have a discounting interval, it generally reduces the discounted failure value more rapidly than the Straight Percent Discounting Method. This results in rapid growth of the reliability estimates. The Standard Discounting Method with the discount interval set at one and the proper choice of the discount fraction will provide nearly identical results to the Lloyd method. For all the reliability growth patterns tested, except low initial reliability with rapid growth, the Lloyd discounting Method caused the model to overestimate the actual reliability. The overestimation was particularly pronounced for low constant reliability where the Lloyd Discounting Method forced the model to overestimate reliability even with γ as large as .99999, Figure 4.7. Because of this characteristic, the Lloyd discounting Method is not recommended for use with the Maximum Likelihood Estimate.

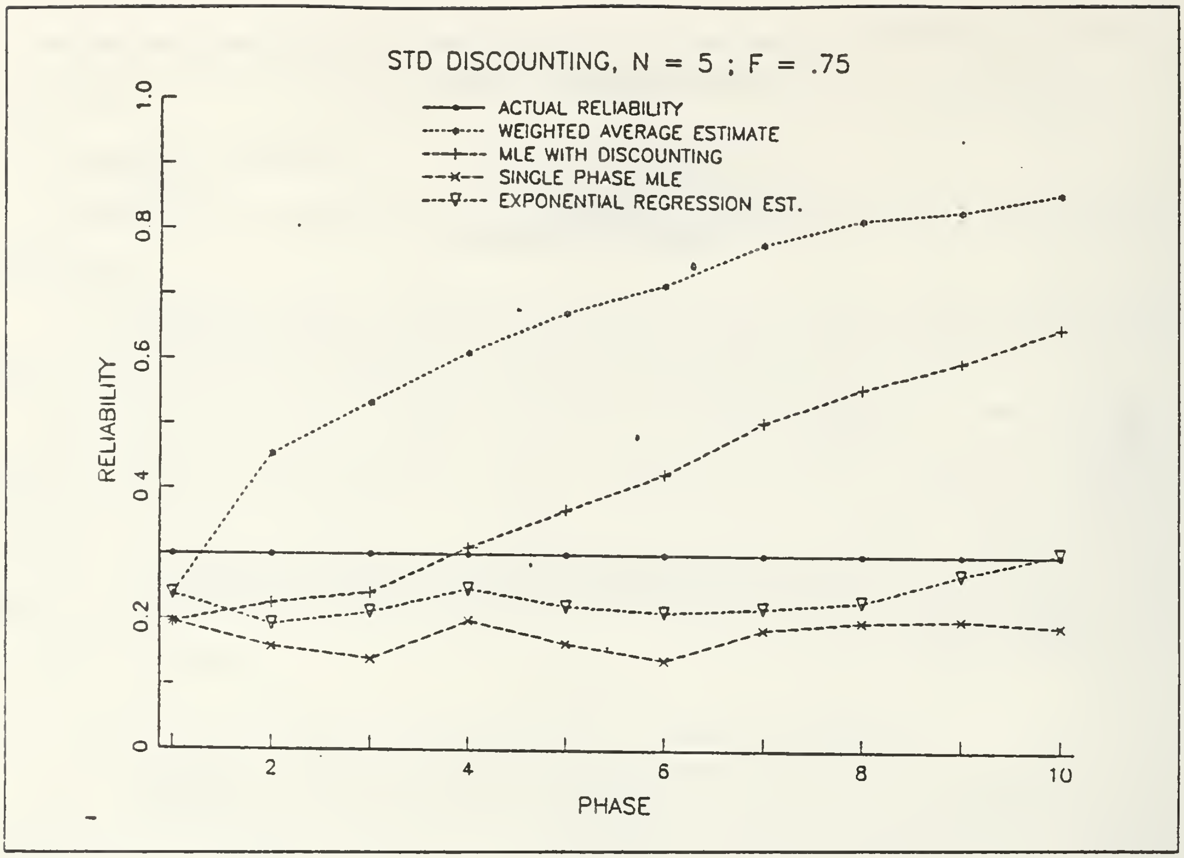


Figure 4.2 Reliability Growth Models
Low Constant Reliability
Std. Discounting Applied.

With the addition of the second parameter, the discount interval, the Straight Percent Discount Method is more flexible than the Lloyd Method. Even so, it has the potential to overestimate the system reliability. The choice of model parameters is critical to the model's performance. Figure 4.8 demonstrates the change in model estimates with varying discounting parameters. Several factors influence the proper choice of parameters. Three of the most important are the amount of testing to be performed, estimates of system reliability before testing begins and how rapidly reliability is expected to grow throughout the development and testing program. No rules have been developed to help select the proper discounting parameters. Experience with the model, good engineering judgement and possibly simulation are necessary to select a combination of discounting parameters that will cause the model to accurately track the actual reliability growth pattern. Even an apparently good choice of failure discounting parameters may result in erroneous reliability estimates if the true

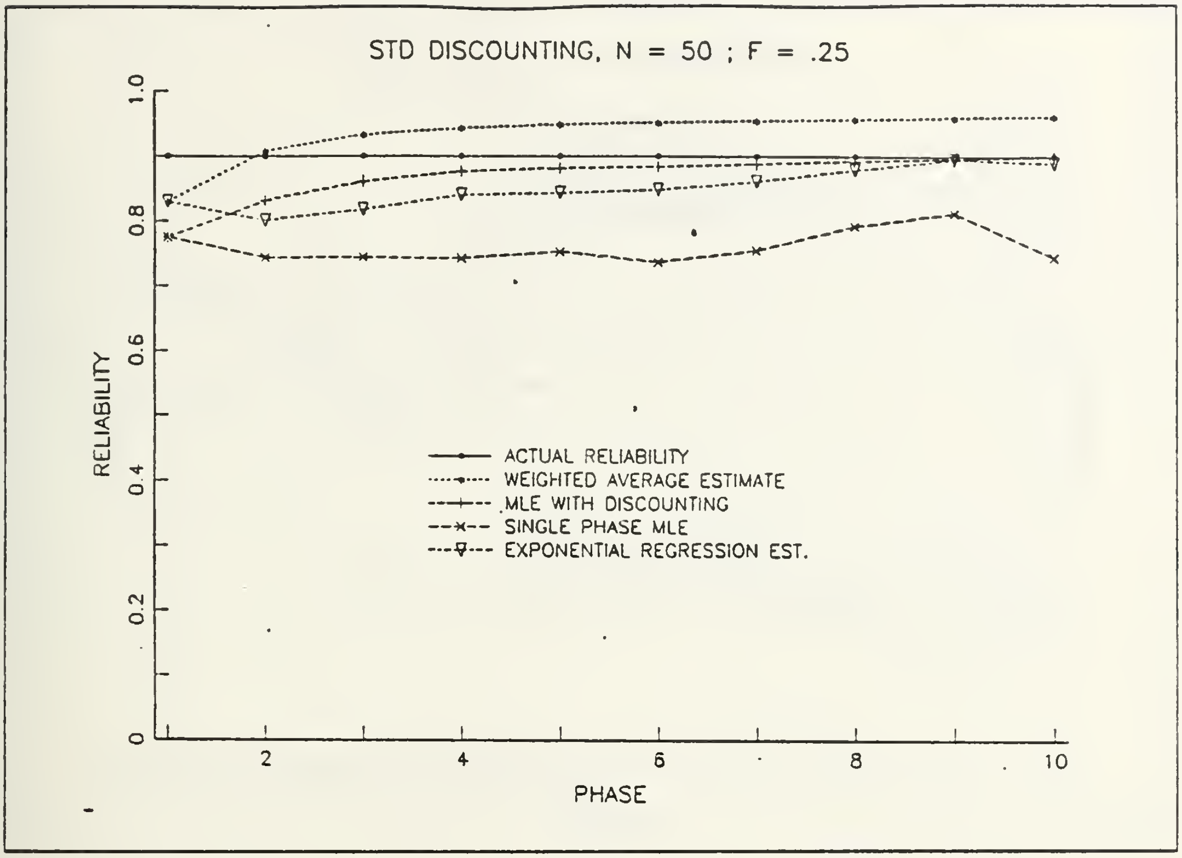


Figure 4.3 Reliability Growth Models
High Constant Reliability
Std. Discounting Applied.

reliability does not grow as expected. Therefore, when using this model, frequent engineering evaluations may be necessary to insure design evolution is progressing as expected.

Reliability estimates from the Maximum Likelihood Estimate With Discounting Model exhibited the smallest variance of the models evaluated. Figures 4.9 and 4.10 are box plots of the one hundred reliability estimates at each testing phase for two of the reliability growth patterns. Comparison with other models may be made from the additional box plots included at Appendix B. The reliability estimates from all three models exhibited large variance in the first few phases of testing. The variance of the estimates gradually decreased as more test data became available. With few exceptions, the MLE With Discounting exhibited the smallest variability over the entire range of phases for each reliability growth pattern tested. This reduced variance was particularly pronounced in the early phases and at low reliability.

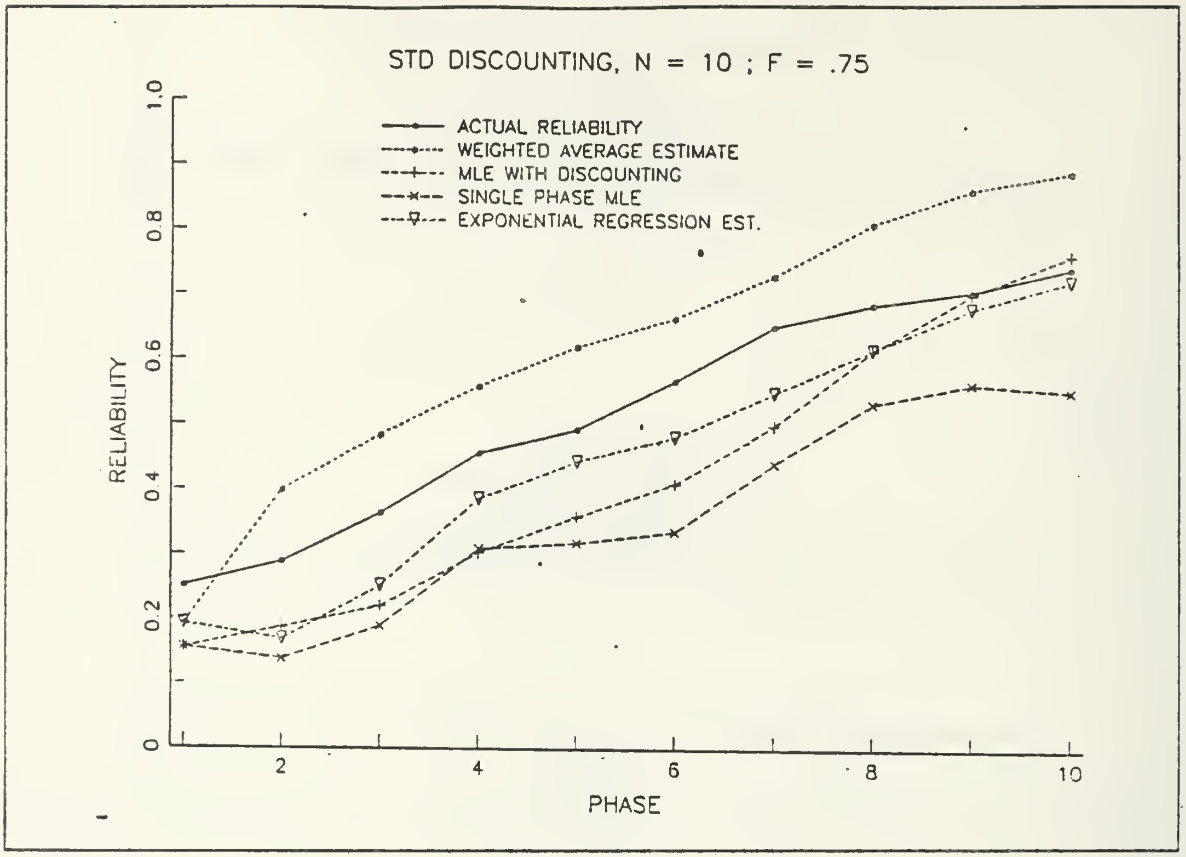


Figure 4.4 Reliability Growth Models
Low Reliability Moderate Growth
Std. Discounting Applied.

The MLE With Discounting Model can accurately track most reliability growth patterns when the Straight Percent Discounting Method is used. It does so with equal or less variance than the other models evaluated. However, the model is very sensitive to the choice of of failure discounting parameters and may either over or under estimate the actual reliability if the parameters are not chosen correctly.

D. EXPONENTIAL REGRESSION MODEL

The Exponential Regression Model has the ability to accurately track most reliability growth patterns. Because it uses a combination of linear regression techniques and failure discounting to track changing reliability, it reacts quickly to changes in actual reliability. Figures 4.2, 4.3, 4.4, 4.5, and 4.6 demonstrate the model's performance with each growth pattern. It also is much less sensitive to the selection of failure discounting parameters than is the MLE With Discounting Model. However,

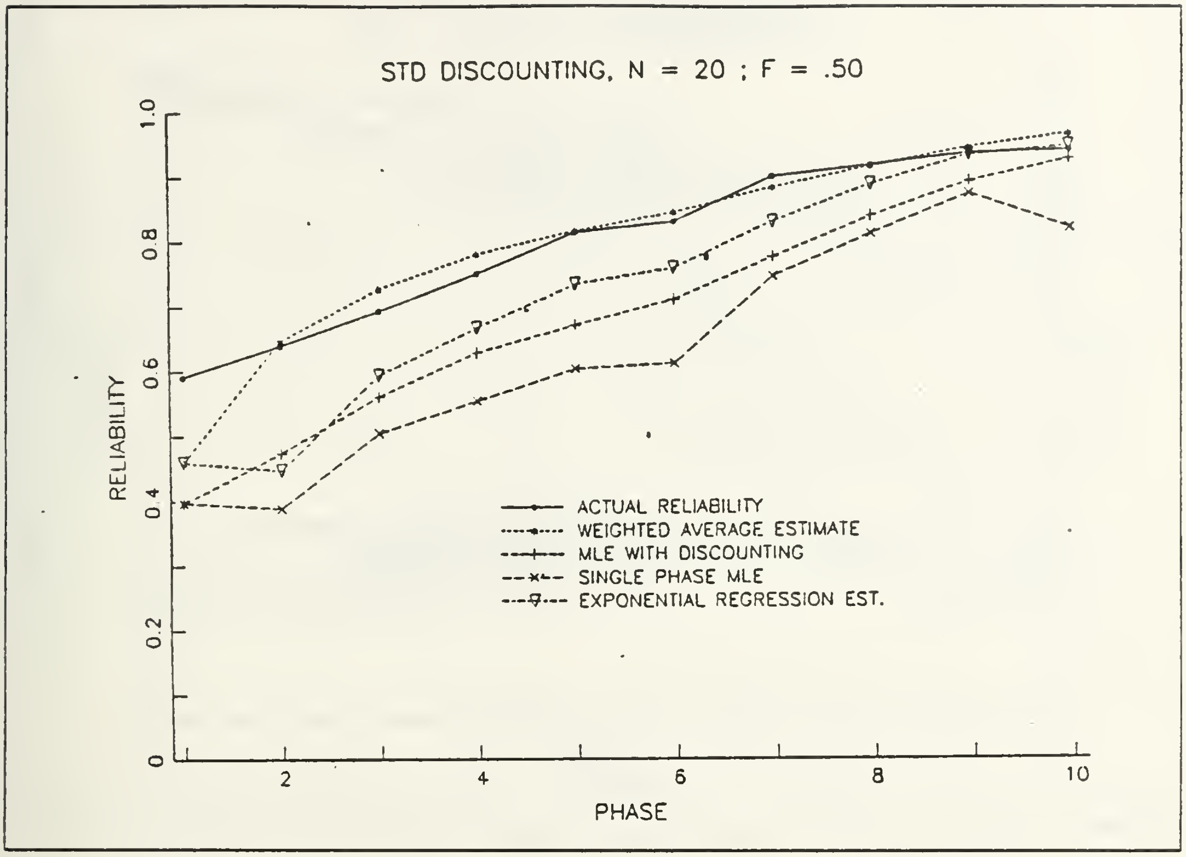


Figure 4.5 Reliability Growth Models
High Reliability Moderate Growth
Std. Discounting Applied.

the variance of the early phase estimates is much greater than the estimates from the MLE With Discounting Model.

This model requires several phases, usually at least four, to accurately estimate system reliability. The number of phases required is a function of the shape of the reliability growth pattern. More phases are required for an accurate estimate at low system reliability than at high reliability. The model also appears to track changing reliability more accurately than constant reliability.

The Exponential Regression Model is relatively robust with respect to the selection of a failure discounting method and parameters. Still, as was the case with the MLE With Discounting Model, the Lloyd Discounting Method tends to cause the model to overestimate actual reliability as is demonstrated by Figure 4.11. Figure 4.12 demonstrates how robust the model can be to changing parameters when the Straight Percent Discount Method is applied. Because the Straight Percent Discounting

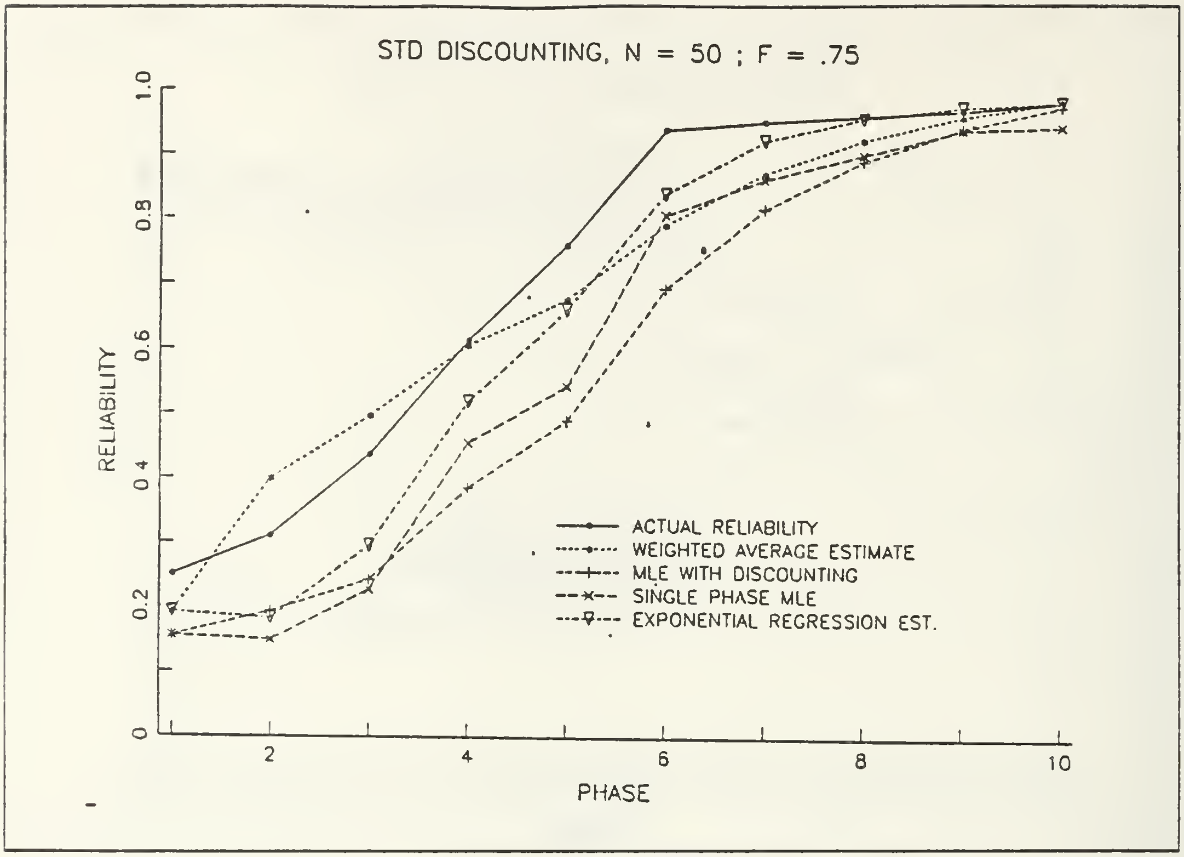


Figure 4.6 Reliability Growth Models
Low Reliability Rapid Growth
Std. Discounting Applied.

Method can produce nearly equivalent results and is much more flexible, use of the Lloyd Discounting Method is not recommended with this model.

Again, no fixed rules exist for chosing the failure discounting parameters of the Straight Percent Discounting Method. Experience with the model, engineering judgement and possibly computer simulation will be necessary to select the proper combination of the discount interval and discount fraction. Even though the model is fairly robust concerning this selection, it is possible to choose parameters that will force the model to overestimate reliability. The danger of this occurring is greatest when expected growth in the system reliability does not occur.

The reliability estimates produced by the exponential regression model exhibit a higher variance than those from the MLE With Discounting Model. This increase in variance is most pronounced in the early testing phases. Figures 4.13 and 4.14 are box plots of the one-hundred reliability estimates at each phase. They demonstrate the variability of the model.

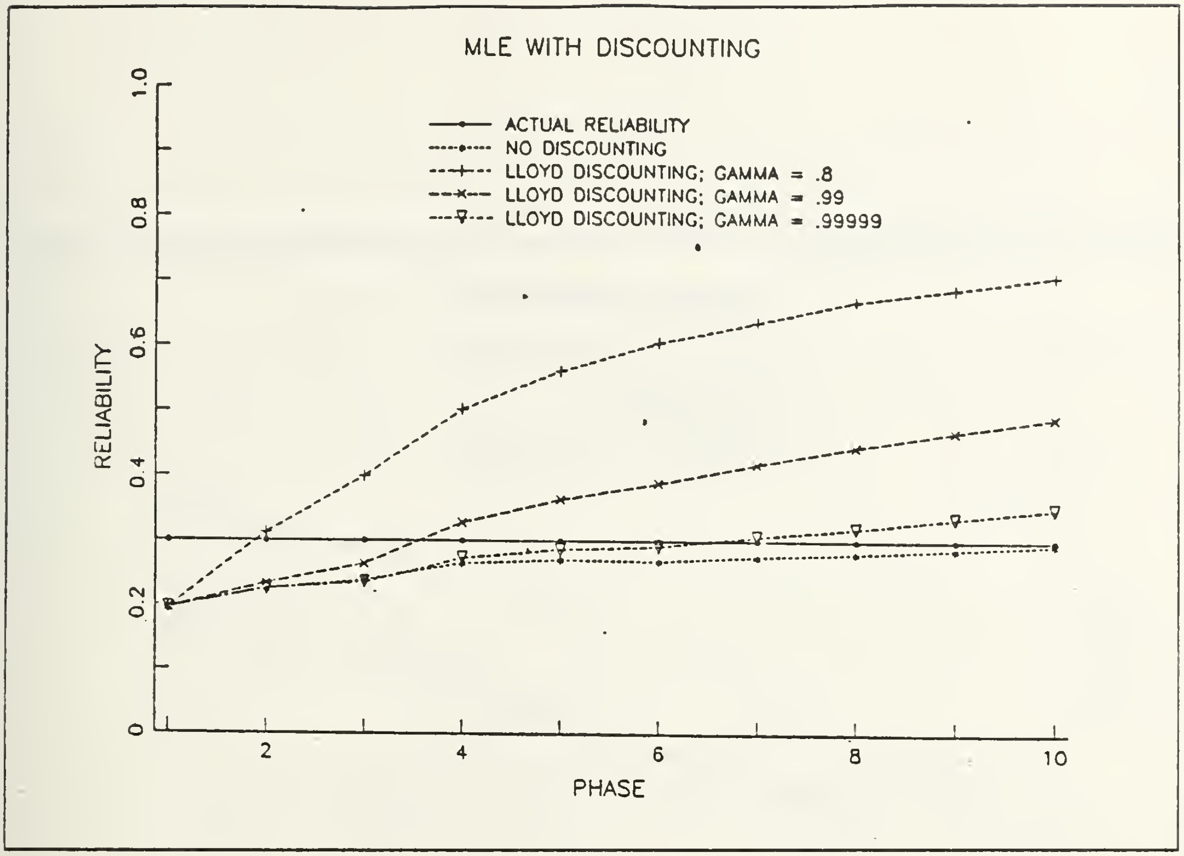


Figure 4.7 MLE With Discounting
Lloyd Discounting Method.

As with any regression, several points are required to stabilize the regression parameter estimates. This causes highly variable responses in the first two or three phases of testing. This variance is largest when reliability in the early phases is low. In later phases and at higher reliability, the variance of estimates from the Exponential Regression Model compares favorably to those of the MLE With Discounting Model.

The Exponential Regression Model closely tracks most reliability growth patterns. It is best when more than three test phases are planned. The most accurate results were obtained when the model was used with the Straight Percent Discounting Method. The model is relatively robust with respect to the selection of discounting parameters; but, may still over or under estimate actual reliability if a bad combination of parameters is selected. The variance of reliability estimates in the early testing phases is large and makes the model undesirable for use with less than four testing phases.

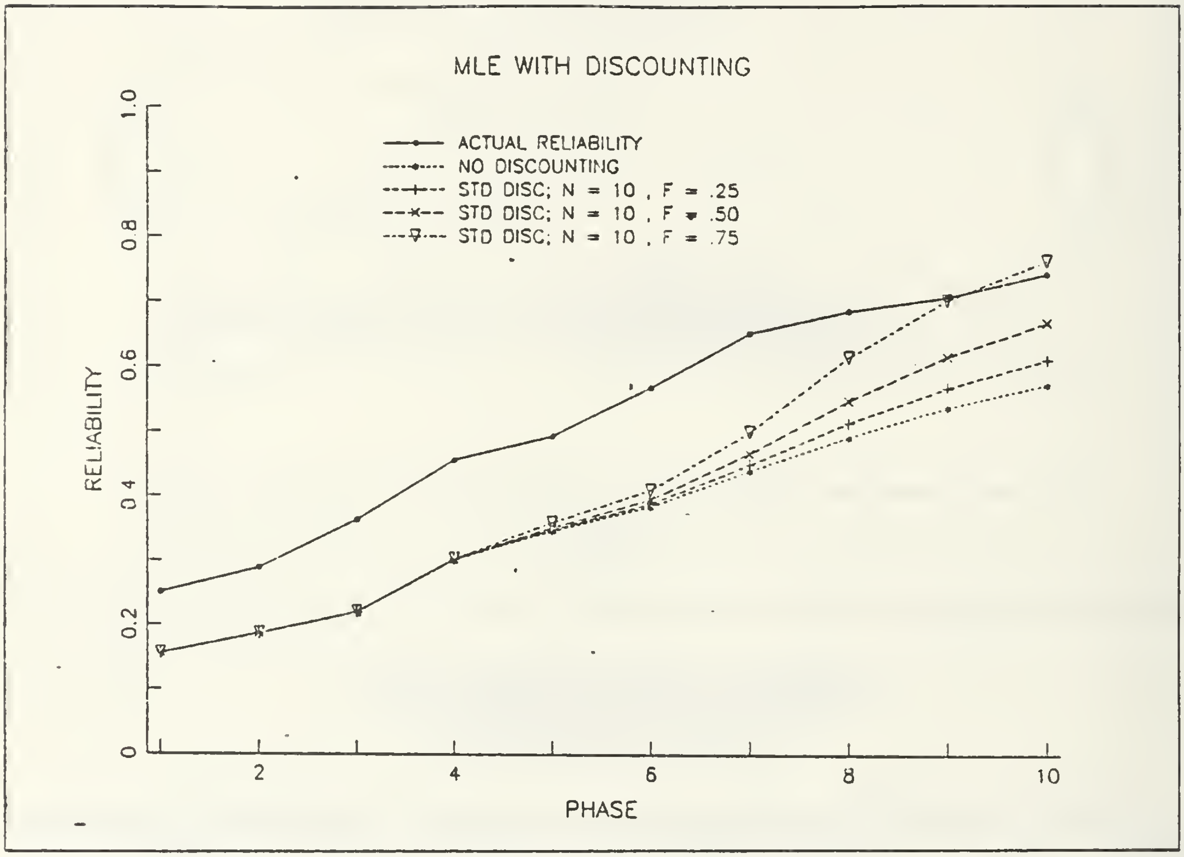


Figure 4.8 MLE With Discounting
Straight Percent Discounting Method.

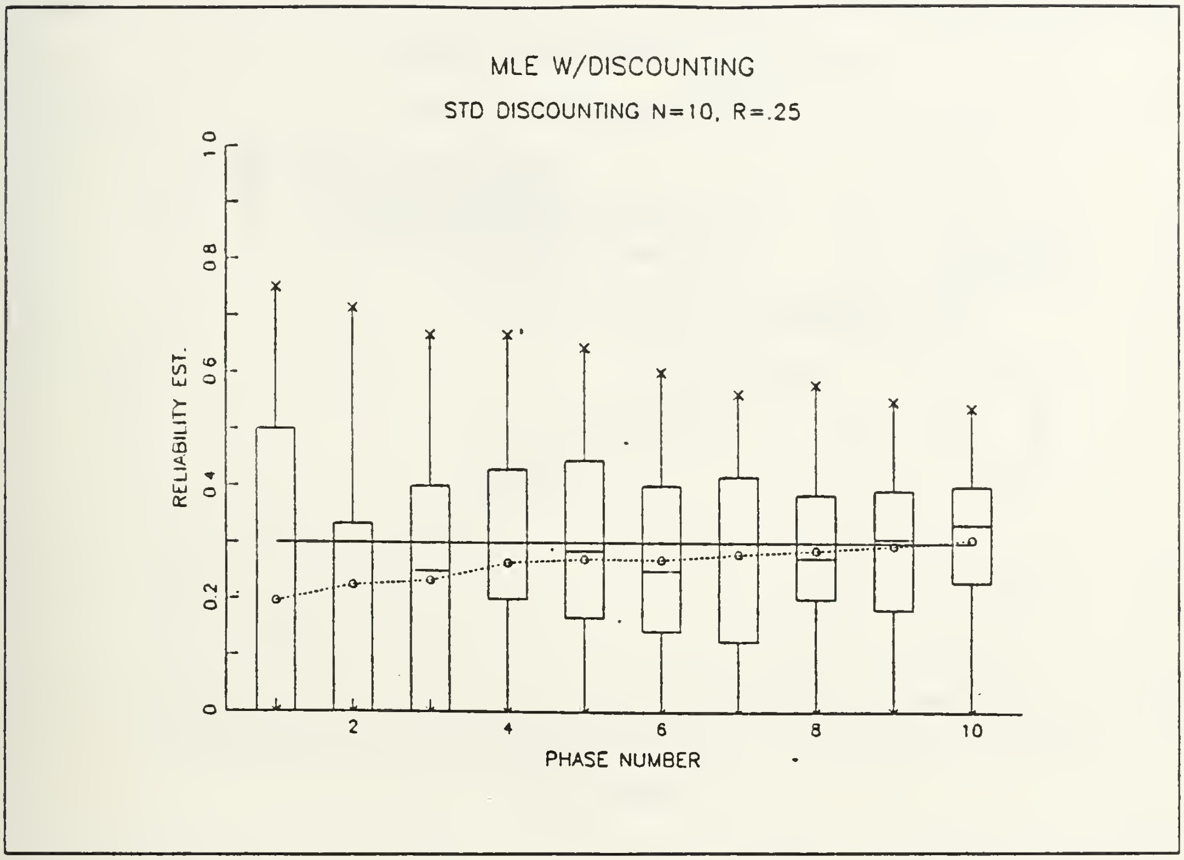


Figure 4.9 MLE With Discounting
Estimate Variability at Low Constant Reliability.

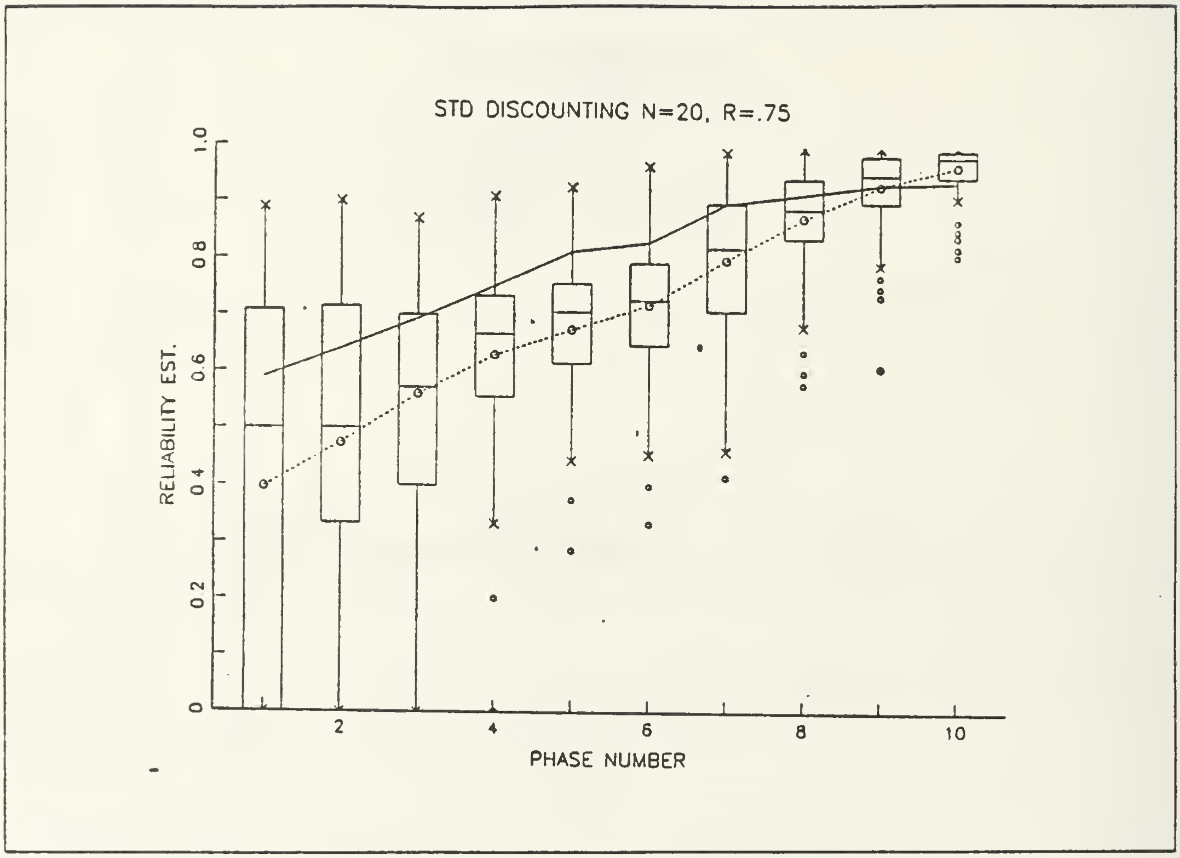


Figure 4.10 MLE With Discounting
Estimate Variability With Increasing Reliability.

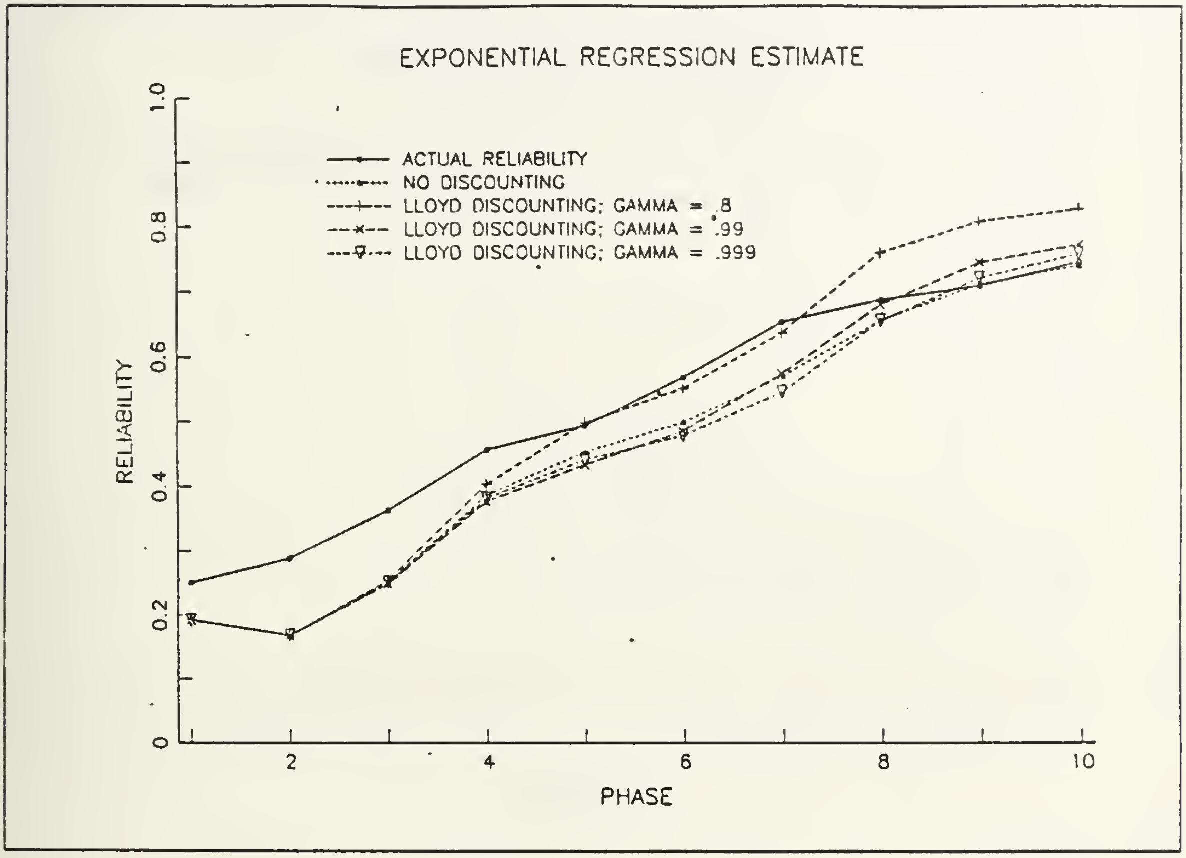


Figure 4.11 Exponential Regression Model
Lloyd Discounting Method.

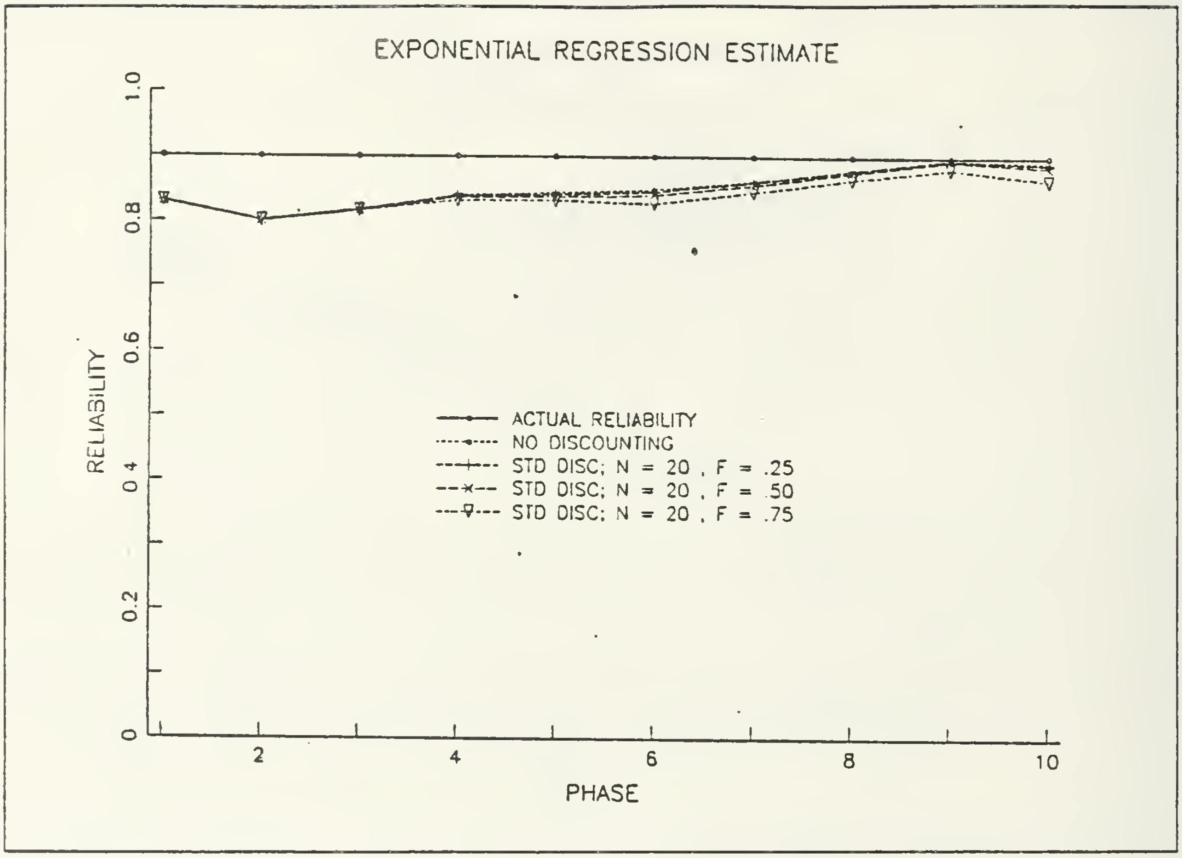


Figure 4.12 Exponential Regression Model
Straight Percent Discounting Method.

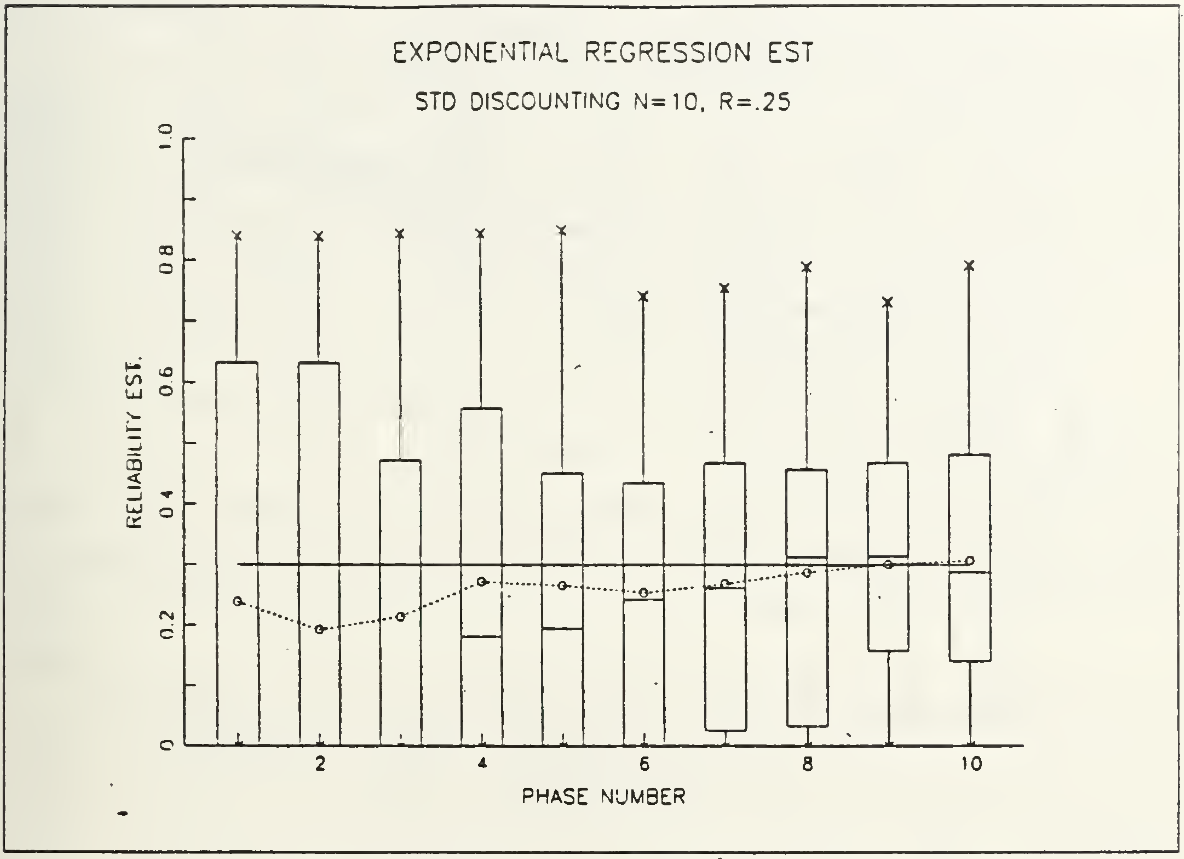


Figure 4.13 Exponential Regression Model Estimate Variability at Low Constant Reliability.

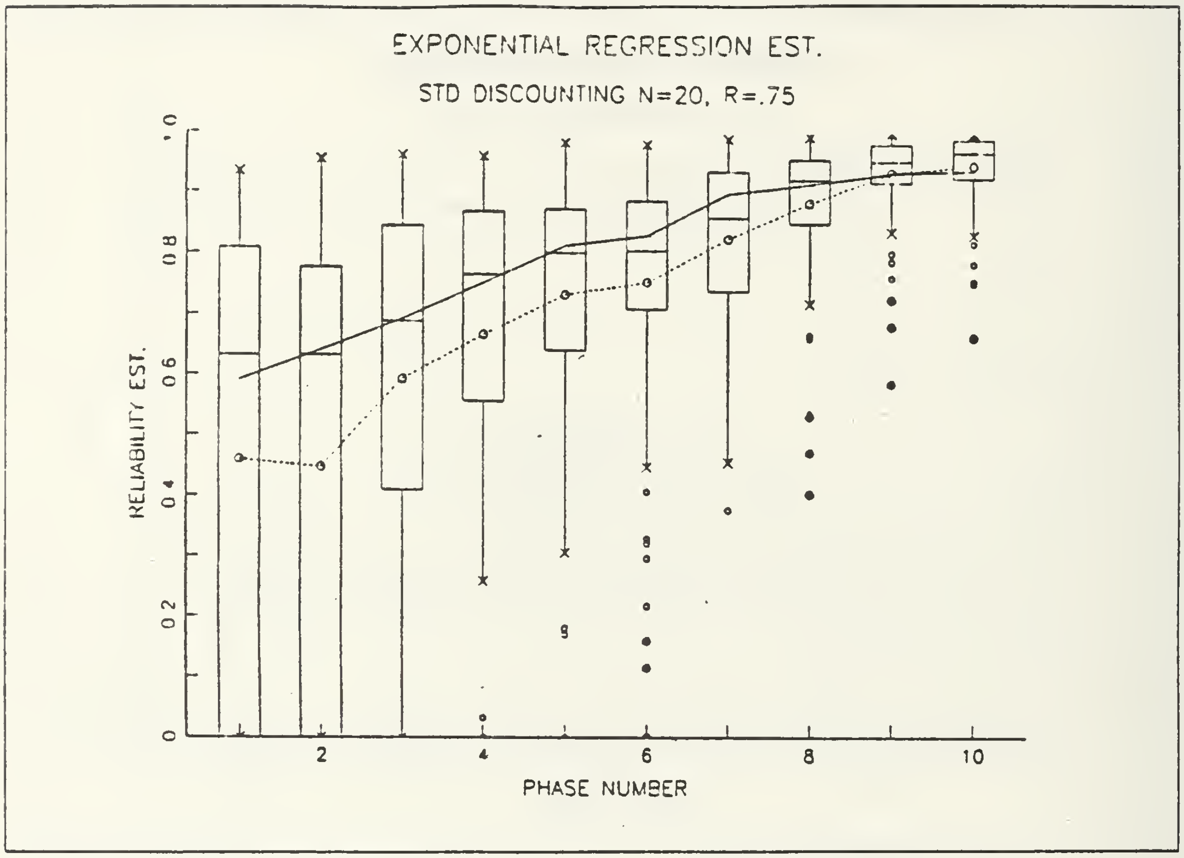


Figure 4.14 Exponential Regression Model Estimate Variability With Increasing Reliability.

V. SUMMARY AND CONCLUSIONS

A. SUMMARY

The purpose of this study is to develop and evaluate discrete reliability growth models that employ fractional failure reduction, referred to as failure discounting. These models use all available data to estimate current reliability even if the actual reliability underlying all test data is not constant. This reduces the resources required to verify system reliability. This study focuses on developmental testing scenarios where testing is terminated after a fixed number of failures have been observed and the results of each trial are recorded as a success or failure, i.e. attribute data.

Failure discounting is performed on past data to make it compatible with test data taken from the current system configuration. Most reliability estimators require a constant actual reliability for each trial to render an accurate estimate. Fractionally discounting past failures adjusts test data from previous testing on less reliable system configurations making the data useful in the computation of the current reliability estimate.

Two different failure discounting methods were used in the study. The first, referred to as the Straight Percent Discounting Method allows the user to predetermine the fraction by which each failure will be reduced, the discount fraction, and an interval between applications of the discount fraction, the discount interval. The discount interval is the number of trials without a reoccurrence of the particular failure cause between application of the discount fraction. The second failure discounting method, referred to as the Lloyd Discounting Method, sets all past failures equal to the upper confidence bound for the probability of reoccurrence for the failure cause. Both failure discounting methods were used with each of the three discrete reliability growth models developed in the study.

Three discrete reliability growth models using failure discounting were developed. The first model adds failure discounting to the maximum likelihood estimate for a proportion. The model is referred to as the Maximum Likelihood Estimate With Discounting Model. To use the model, the failure discounting method of choice is applied to the test data. The resulting discounted failures are returned to unity by adjusting the number of successful trials preceding each failure. The estimate of system

reliability is computed as the ratio of adjusted successful trials to adjusted total trials. Failure discounting allows the model to track changing reliability.

The remaining two models are modifications of an exponential single phase reliability estimate of the form reliability = $1 - e^{A}$. One model uses regression to estimate A. This model is referred to as the Exponential Regression Model. The second model uses an unbiased estimate of A to produce a single testing phase reliability estimate. The current reliability estimate is a weighted average of the previous testing phase reliability estimate and the current single phase reliability estimate. The fraction of the total test trials to date used in the computation of each estimate becomes the weighting factor for the arithematic average. As with the Maximum Liklihood With Discounting Model, a failure discounting method is applied to the test data before either model is used to compute a reliability estimate. Both these models have the ability to track changing reliability without the use of failure discounting; but, failure discounting is employed to enhance the models performance.

A Monte-Carlo simulation was written to evaluate the performance of the reliability growth models and failure discounting methods developed in the study. This was necessary because closed form solutions for the statistical properties of the models are not mathematically tractable. The simulation uses input parameters to generate a reliability growth pattern. Test data including the number of trials up to and including system failure and the cause of system failure are generated from the established reliability growth pattern. Each of the three reliability growth are applied to the simulated test data resulting in an estimate of system reliability at each testing phase. After this procedure is replicated a suitable number of times, estimates for the mean and standard deviation of each estimate are computed. These estimates are used to evaluate the performance of the reliability growth models. By varying the reliability growth pattern and the failure discounting parameters, the models performance over a wide range of situations may be evaluated. The conclusions reached from this analysis are included in the following section.

B. CONCLUSIONS

Even though the analysis of the reliability growth models performed in the study is not comprehensive enough to state conclusions with certainty, several general conclusions are apparent.

The Lloyd Failure Discounting Method should not be used with the three reliability growth models developed in the study. With few exceptions, this method

caused the growth models to overestimate system reliability. This tendency to overestimate was particularly evident at low reliabilities when little or no actual reliability growth occurred. The few situations where the method performed well, rapid reliability growth, were handled equally well by the Straight Percent Discounting Method. The Straight Percent Discounting Method's flexibility makes the preferred of the two methods for fractionally discounting failures.

The Weighted Average Reliability Growth Model, in its current form, should not be used to estimate system reliability. The model grossly overestimated reliability for most growth patterns tested. Even when no failure discounting was applied, the model overestimated system reliability. Adding either failure discounting method only increased the tendency to overestimate.

Both the Maximum Liklihood Estimate With Discounting and the Exponential Regression Model exhibited performance superior to the standard, single phase MLE, estimate. But there is no clear cut choice between the Maximum Likelihood Estimate With Failure Discounting Model and the Exponential Regression Model. Both models accurately track a wide range of reliability growth patterns. The MLE With Discounting Model appears to be the less variable of the two methods but is much more sensitive to the choice of failure discounting parameters than is the Exponential Regression Model. The variability of both models decreased with increasing reliability and additional test data. For this reason the Exponential Regression Model is recommended for situations where four or more testing phases are planned and the system possesses high reliability with near certainty. For other cases the Maximum Likelihood Estimate With Failure Discounting is recommended because of its reduced variability.

The discrete reliability growth models using failure discounting developed in the study represent an improvement over conventional reliability estimation techniques for the special case of testing until a fixed number of failures have been observed with attribute data. However, these models can be improved. The following section contains recomendations for improvements and further study.

C. RECOMENDATIONS

The following are recomendations for improvement to the models introduced in this study.

- Many more simulation runs with varied reliability growth patterns and failure discounting parameters are required before solid conclusions concerning the properties of the reliability growth models can be reached.

- The Monte-Carlo Simulation should be modified so declining reliability can be modeled.
- An alternate weighting technique should be developed for the Weighted Average Model to correct its tendency to overestimate reliability. In certain situations, this model demonstrated the smallest variability of the three models developed. The model might prove to be an excellent reliability estimator if the overestimation tendency can be corrected.
- An additional failure discounting method that combines the flexibility of the Straight Percent Discounting Method with the non-arbitrary selection of the discounting fraction from the Lloyd method might prove useful. This could be accomplished by adding a discount interval to the current Lloyd Discounting Method.
- The possibility of using a simulation similar to the one developed here to aid in the selection of failure discounting parameters should be explored. Using an estimate of initial reliability, a reliability goal and the number of planned trials; the various choices for failure discounting parameters could be tested and one set chosen based on the simulation results.
- Further analysis of the performance of the reliability growth models with varying failure discounting parameters is required to develop "rules of thumb" for failure parameter selection.

APPENDIX A

DISCRETE RELIABILITY GROWTH MODEL SIMULATION

1. FORTRAN CODE FOR THE SIMULATION

```
************************************************************************
*                                                                      *
*          DISCRETE RELIABILITY GROWTH SIMULATION                      *
*                                                                      *
*              PROGRAMMED BY JAMES E DRAKE                             *
*                                                                      *
*               LAST MODIFIED 19 AUG 1987                              *
*                                                                      *
*  THE FOLLOWING EXTERNAL FILES ARE USED BY THE PROGRAM                *
*  INPUT  : DATA AND PARAMETER INPUT FILE (DEVICE # 10)                *
*  THESIS : OUTPUT FILE CONTAINING INTERMEDIATE COMPUTATIONS           *
*           (DEVICE # 20)                                              *
*  RELIAB: OUTPUT FILE CONTAINING FINAL RESULTS OF THE SIMULATION      *
*           (DEVICE # 30)                                              *
*  EST    : OUTPUT FILE CONTAINING EACH PHASE ESTIMATE FOR EACH        *
*           REPLICATION OF THE WEIGHTED AVERAGE ESTIMATE               *
*           (DEVICE # 40)                                              *
*  MLEWD  : OUTPUT FILE CONTAINING MLE ESTIMATES USING DISCOUNTING     *
*           FOR EACH PHASE AND EACH REPLICATION                        *
*           (DEVICE # 50)                                              *
*  MLESP  : OUTPUT FILE CONTAINING MLE ESTIMATE FOR EACH SINGLE PHASE  *
*           AND ALL REPLICATIONS USING NO DISCOUNTING                  *
*           (DEVICE # 60)                                              *
*  REGEST : OUTPUT FILE CONTAINING EACH PHASE ESTIMATE FOR EACH        *
*           REPLICATION OF THE EXPONENTIAL REGRESSION ESTIMATE         *
*           (DEVICE # 70)                                              *
*                                                                      *
*  THE FOLLOWING IS A LIST OF KEY ARRAYS USED IN THE SIMULATION        *
*                                                                      *
*  A      : MAIN WORKING ARRAY CONTAINS PROBABILITY OF SUCCESS FOR     *
*           EACH FAILURE CAUSE, NUMBER OF TRIALS UNTIL FAILURE FOR     *
*           EACH FAILURE CAUSE AND THE SYSTEM, CAUSE OF FAILURE,       *
*           PHASE NUMBER, ADJUSTED NUMBER OF TRIALS AND ADJUSTED       *
*           NUMBER OF FAILURES                                         *
*           DIMENSION ( ((2*#CAUSES)+7),#FAILURES )                    *
*  NFAPH  : CONTAINS THE NUMBER OF FAILURES IN EACH PHASE              *
*           DIMENSION (1,#PHASES)                                      *
*  NFCAUS : BINARY ARRAY USED TO DETERMINE IF A FAILURE OCCURRED IN    *
*           A PHASE                                                    *
*           DIMENSION ( 1,#FAILURE CAUSES)                             *
*  NTRIAL : CONTAINS THE NUMBER OF TRIALS SINCE LAST FAILURE OR        *
*           DISCOUNTING FOR EACH FAILURE CAUSE                         *
*           DIMENSION ( 1,#FAILURE CAUSES )                            *
*  PHREST : RECORDS THE PHASE ESTIMATE FOR EACH ESTIMATOR WITHIN A     *
*           SINGLE REPLICATION                                         *
*           DIMENSION (4,#PHASES)                                      *
*           ROW 1 : WEIGHTED AVERAGE ESTIMATE                          *
*           ROW 2 : MLE WITH DISCOUNTING                               *
*           ROW 3 : SINGLE PHASE MLE                                   *
*           ROW 4 : EXPONENTIAL REGRESSION ESTIMATE                    *
*  AREL   : CONTAINS ACTUAL SYSTEM RELIABILITY IN EACH PHASE           *
*           DIMENSION (1,#PHASES)                                      *
*  YJK    : CONTAINS YJK VALUES UP TO 1000                             *
*           DIMENSION (1,1000)                                         *
*  CUMSF  : CONTAINS THE NUMBER OF SUCCESS AND FAILURES FOR EACH       *
*           FAILURE CAUSE (USED WITH WEIGHTED AVERAGE EST.)            *
*           DIMENSION (3,#FAILURE CAUSES)                              *
*           ROW 1 : NUMBER OF FAILURES                                 *
*           ROW 2 : NUMBER OF SUCCESSES                                *
```

```
*          ROW 3 : ADJUSTED NUMBER OF SUCCESSES                           *
*  REG    : ARRAY USED TO COMPUTE THE EXPONENTIAL REGRESSION ESTIMATE     *
*          DIMENSION (5,#PHASES)                                          *
*          ROW 1 : K BAR                                                  *
*          ROW 2 : Y BAR                                                  *
*          ROW 3 : Y BAR FOR THE PHASE                                    *
*          ROW 4 : B HAT                                                  *
*          ROW 5 : A HAT                                                  *
*                                                                         *
*  THE REMAINING ARRAYS ARE USED TO COMPUTE THE MEAN AND VARIANCE         *
*  OF EACH ESTIMATE AT EACH PHASE. THEY ALL HAVE THE SAME DIMENSIONS      *
*  AND STRUCTURE                                                          *
*          DIMENSION (4,#PHASES)                                          *
*          ROW 1 : RUNNING SUM OF ESTIMATES                               *
*          ROW 2 : RUNNING SUM OF SQUARED ESTIMATES                       *
*          ROW 3 : MEAN OF THE ESTIMATES                                  *
*          ROW 4 : STANDARD DEVIATION OF THE ESTIMATES                    *
*                                                                         *
*  EST    : VALUES FOR THE WEIGHTED AVERAGE ESTIMATE                      *
*  MLEWD  : VALUES FOR THE MLE WITH DISCOUNTING                           *
*  MLESP  : VALUES FOR THE SINGLE PHASE MLE                               *
*  REGEST : VALUES FOR THE EXPONENTIAL REGRESSION ESTIMATE                *
*                                                                         *
***************************************************************************


C  DEFINE AND DIMENSION VARIABLES
      PARAMETER (NR=50,NC=200)
      INTEGER REP,CUMSF,DISOPT
      REAL*4 MIN
      REAL*8 DSEED,MLESP,MLEWD,EST
      DIMENSION NFAPH(NR),A(NR,NC),NFCAUS(NR),NTRIAL(NR),PHREST(4,NR),ES
     CT(4,NR),MLEWD(4,NR),MLESP(4,NR),REGEST(4,NR),AREL(NR),YJK(1000),CU
     CMSF(3,NR),REG(5,NR)
C  READ IN THE NUMBER OF CAUSES TO BE USED ( NCAUSE ) AND THE NUMBER
C  OF PHASES ( NPHASE ) IN THE TEST
      READ(10,*) NCAUSE
      READ(10,*) NPHASE
C  CREATE VARIABLES FOR THE ROW INDICES OF THE WORKING MATRIX ( A )
C    IPHASE: PHASE
C    ISYSPR: ACTUAL COMPONENT RELIABILITY
C    INTR: NUMBER OF TRIALS UP TO AND INCLUDING FAILURE
C    IFAILC: CAUSE OF THE FAILURE
C    IADJF: ADJUSTED NUMBER OF FAILURES                                  ED
C           AFTER DISCOUNTING HAS BEEN APPLIED
C    IADJT: ADJUSTED NUMBER OF TRIALS AFTER DISCOUNTING HAS BEEN APPLIED
C    IYJK: YJK COMPUTED ON THE ADJUSTED NUMBER OF TRIALS
      IPHASE = (2*NCAUSE)+1
      ISYSPR = IPHASE +1
      INTR = ISYSPR + 1
      IFAILC = INTR + 1
      IADJF = IFAILC + 1
      IADJT = IADJF + 1
      IYJK = IADJT + 1

C  READ IN THE NUMBER OF FAILURES IN EACH PHASE ( NFAPH(I) ) AND
C  COMPUTE THE TOTAL NUMBER OF FAILURES IN THE TEST ( NFAIL )
      NFAIL = 0
      DO 10 I=1,NPHASE
            READ(10,*) NFAPH(I)
            NFAIL = NFAIL + NFAPH(I)
 10   CONTINUE
C  INPUT THE PROBABILITY OF SUCCESS IN A SINGLE TRIAL FOR EACH CAUSE
C  IN THE FIRST PHASE
```

```
      DO 20 I=1,NCAUSE
           READ(10,*) A(I,1)
 20   CONTINUE
C  INPUT THE REMAINING VARIABLES , THE NUMBER OF SUCCESSFUL TRIALS
C  BEFORE A DISCOUNT IS APPLIED (N); THE DISCOUNT FACTOR (R); THE SEED
C  FOR THE RANDOM NUMBER GENERATOR, GGUBFS, (DSEED); RELIABILITY
C  GROWTH FRACTION (FRIMP); TRIGGER FOR PRINTING INTERMEDIATE OUTPUT
C  (IOPT)
C  TRIGGERS FOR SAVING EACH ESTIMATE AT EACH PHASE FOR EACH ESTIMATOR
C     IOPT1 : WEIGHTED AVERAGE MODEL
C     IOPT2 : MLE WITH DISCOUNTING
C     IOPT3 : SINGLE PHASE MLE
C     IOPT4 : EXPONENTIAL REGRESSION MODEL
C   DISCOUNTING OPTION TRIGGER (DISOPT); LLOYDS FAILURE DISCOUNTING
C   PARAMETER (GAMMA)
      READ(10,*) N
      READ(10,*) R
      READ(10,*) DSEED
      READ(10,*) FRIMP
      READ(10,*) NREP
      READ(10,*) IOPT
      READ(10,*) IOPT1
      READ(10,*) IOPT2
      READ(10,*) IOPT3
      READ(10,*) IOPT4
      READ(10,*) DISOPT
      READ(10,*) GAMA
      XNREP = NREP
      DSEED1 = DSEED
C  INITIALIZE THE ARRAYS USED TO COMPUTE THE MEAN AND STANDARD DEVIATION
C  OF EACH ESTIMATOR
      DO 30 J=1,NPHASE
           DO 30 I=1,4
                EST(I,J) = 0.0
                MLEWD(I,J) = 0.0
                MLESP(I,J) = 0.0
                REGEST(I,J) = 0.0
                PHREST(I,J) = 0.0
 30   CONTINUE
C  COMPUTE AND STORE THE YJK VALUES UP TO 1000
      YJK(1) = 0.0
      DO 40 I=1,999
           YJK(I+1) = YJK(I) + 1.0/I
 40   CONTINUE
C  COMPUTE AND STORE K BAR FOR THE EXPONENTIAL REGRESSION MODEL
      SUM = 0.0
      DO 50 I=1,NPHASE
         SUM = SUM + I
         REG(1,I) = SUM/I
 50   CONTINUE
C  MAJOR REPETITION OF THE SIMULATION LOOP
      DO 500 REP=1,NREP

C  INITIALIZE FAILURE CAUSE VECTOR (NFCAUS) AND (CUMSF)
C  COMPUTE THE INITIAL SYSTEM RELIABILITY
      REL = 1.
      DO 60 I=1,NCAUSE
           NFCAUS(I) = 0
           REL = REL * A(I,1)
           DO 60 J=1,3
              CUMSF(J,I) = 0
 60   CONTINUE
```

```
C  INITIALIZE COLUMN (FAILURE # ) COUNTER FOR THE WORKING ARRAY (A)
      J = 1
C  LOOP TO COMPUTE THE NUMBER OF TRIALS UP TO AND INCLUDING FAILURE
C  AND THE CAUSE OF FAILURE FOR EACH FAILURE IN EACH PHASE
      DO 130 K=1,NPHASE
C  SKIP ACTUAL COMPONENT RELIABILITY COMPUTATION AFTER FIRST REP
C  AND FOR FIRST FAILURE
        IF(J.EQ.1) GOTO 75
        IF(REP.GT.1) GOTO 75
        REL = 1.
C  COMPUTE NEW ACTUAL RELIABILITY FOR THE COMPONENT IN PHASE K
        DO 70 I=1,NCAUSE
C  INCREASE CAUSE PR(SUCCESS) IF IT CAUSED FAILURE IN THE PREVIOUS PHASE
C  COMPUTE NEXT PHASE RELIABILITY AND REINITIALIZE NFCAUS
          IF(NFCAUS(I).EQ.1) THEN
          A(I,J) = A(I,(J-1)) + ((1. - A(I,(J-1)))*FRIMP)
          ELSEIF(NFCAUS(I).NE.1) THEN
          A(I,J) = A(I,(J-1))
          ELSE
          ENDIF
          REL = REL*A(I,J)
          NFCAUS(I) = 0
 70     CONTINUE
 75     J1 = 1
        TRTOT = 0.0
C  COMPUTE THE NUMBER OF TRIALS UP TO AND INCLUDING FAILURE AND THE
C  CAUSE OF FAILURE FOR EACH FAILURE IN THE PHASE
        DO 120 L=1,NFAPH(K)
          IF(REP.GT.1) GOTO 90
          IF(J1.EQ.1) GOTO 85
          DO 80 I=1,NCAUSE
            A(I,J) = A(I,(J-1))
 80       CONTINUE
 85       A(ISYSPR,J) = REL
          A(IPHASE,J) = K
 90       MIN = 7.2E75
          DO 110 I=1,NCAUSE
C  ASSIGN # TRIALS FOR CAUSES WITH PR(SUCCESS) = 0 OR 1
            IF(A(I,J).GE.1.) THEN
              A((I+NCAUSE),J) = 7.2E75
              GOTO 100
            ELSEIF(A(I,J).EQ.0.) THEN
              A((I+NCAUSE),J) = 1.
              GOTO 100
            ELSE
            ENDIF
C  CONVERT UNIFORM (0,1) RANDOM VARIABLE TO GEOMETRIC (# TRIALS UNTIL
C  FAILURE ) FOR EACH FAILURE CAUSE. RECORD THE MIN # TRIALS FOR THE
C  CAUSES AS THE SYSTEM # TRIALS UP TO AND INCLUDING FAILURE AND
C  RECORD THE FAILURE CAUSE
            A((I+NCAUSE),J) = INT(1.+(LOG(GGUBFS(DSEED))/LOG(A(I,J))))
 100        IF(A((I+NCAUSE),J).LE.MIN) THEN
              MIN = A((I+NCAUSE),J)
              IMIN = I
            ELSE
            ENDIF
 110      CONTINUE
          A(IFAILC,J) = IMIN
          NFCAUS(IMIN) = 1
C  COMPUTE THE TOTAL # OF TRIALS FOR THE MLE SINGLE PHASE ESTIMATE AND
C  INCREMENT FAILURE # COUNTERS
```

```
          A(INTR,J) = MIN
          TRTOT = TRTOT + A(INTR,J)
          J = J + 1
          J1 = J1 + 1
 120  CONTINUE
C  COMPUTE THE MLE ESTIMATE OF COMPONENT RELIABILITY FOR THIS PHASE AND
C  COMPUTE THE RUNNING SUM OF ESTIMATES AND THE SUM OF ESTIMATES SQUARED
C  FOR COMPUTATION OF THE MEAN AND STANDARD DEVIATION OF THE ESTIMATE
        PHREST(3,K) = (TRTOT - NFAPH(K))/TRTOT
        MLESP(1,K) = MLESP(1,K) + PHREST(3,K)
        MLESP(2,K) = MLESP(2,K) + (PHREST(3,K)**2)
 130  CONTINUE
C  INITIALIZE THE ADJUSTED NUMBER OF FAILURES TO 1 AND THE COUNT OF THE
C  NUMBER OF TRIALS SINCE FAILURE OR DISCOUNTING (NTRIALS(I) ) TO 0
C  IN PREPARATION FOR THE DISCOUNTING ROUTINE
        DO 140 J=1,NFAIL
           A(IADJF,J) = 1.
 140  CONTINUE
        DO 150 I=1,NCAUSE
           NTRIAL(I) = 0
 150  CONTINUE
C  DISCOUNTING ROUTINE REVIEWS ALL PAST FAILURES AND CAUSES TO DATE
C  AND DETERMINES IF THE DISCOUNTING CONDITIONS HAVE BEEN MET. COMPUTES
C  THE ADJUSTED FAILURES, THE ADJUSTED # OF TRIALS AND YJK
        J = 0
        DO 300 K=1,NPHASE
          DO 200 L=1,NFAPH(K)
            J = J + 1
C  UPDATES THE NUMBER OF TRIALS SINCE FAILURE OR DISCOUNTING FOR EACH
C  FAILURE CAUSE
            ICAUSE = INT(A(IFAILC,J)+.5)
            DO 160 I=1,NCAUSE
              IF(ICAUSE.EQ.I) THEN
                NTRIAL(I) = 0
              ELSEIF(ICAUSE.NE.I) THEN
                NTRIAL(I) = NTRIAL(I) + INT(A(INTR,J)+.5)
              ELSE
              ENDIF
 160     CONTINUE
 200  CONTINUE
C  CHOOSE DISCOUNTING METHOD TO BE USED
          IF(DISOPT.NE.2) GOTO 180
C  PERFORM LLOYDS FAILURE DISCOUNTING METHOD
          DO 170 I=1,J
                I1 = INT(A(IFAILC,I)+.5)
                IF(NTRIAL(I1).EQ.0) THEN
                      A(IADJF,I) = 1.0
                      GOTO 170
                ELSE
                ENDIF

                A(IADJF,I) = 1.0 - ((1.-GAMA)**(1.0/NTRIAL(I1)))
 170     CONTINUE
          GOTO 210
C  PERFORMS STRAIGHT PERCENT FAILURE DISCOUNTING AND
C  COMPUTES THE ADJUSTED # OF FAILURES
 180     DO 190 I=1,J
              I1 = INT(A(IFAILC,I)+.5)
              IF(NTRIAL(I1).EQ.0) THEN
                A(IADJF,I) = 1.
              ELSEIF(NTRIAL(I1).GE.N) THEN
```

```
              A(IADJF,I) = A(IADJF,I)*((1.-R)**(NTRIAL(I1)/N))
           ELSE
           ENDIF
 190      CONTINUE
C  ADJUSTS THE # TRIALS SINCE FAILURE OR DISCOUNTING FOR THOSE CAUSES
C  THAT HAVE MET OR SURPASSED THE DISCOUNTING THRESHOLD
C  FOR THE STRAIGHT PERCENT DISCOUNTING METHOD
         DO 205 I=1,NCAUSE
           IF(NTRIAL(I).GE.N) NTRIAL(I) = MOD(NTRIAL(I),N)
 205      CONTINUE
 210    TADJT = 0.0
        TYJK = 0.0
        TPYJK = 0.0
        K1 = 0
        DO 215 I2=1,3
        DO 215 I=1,NCAUSE
          CUMSF(I2,I) = 0
 215      CONTINUE
C  COMPUTES THE ADJUSTED # OF TRIALS FROM THE ADJUSTED # OF FAILURES
C  AND COMPUTES THE SUM OF THE ADJUSTED # OF TRIALS FOR ESTIMATE COMP.
        PREL = 0.0
        LTRIAL = 0
        DO 240 I=1,J

         A(IADJT,I) = A(INTR,I)/A(IADJF,I)
         TADJT = TADJT + A(IADJT,I)
C  COMPUTE YJK FROM THE ADJUSTED # OF TRIALS AND STORE THE SUM FOR
C  ESTIMATE COMPUTATION, USE ARRAY FOR # TRIALS < 1000 AND APPROX. FOR
C  VALUES > 1000
         N1 = NINT(A(IADJT,I))
         IF(N1.LE.1000) THEN
              A(IYJK,I) = YJK(N1)
         ELSEIF(N1.GT.1000) THEN
              A(IYJK,I) = (A(IADJT,I)**.10995)*3.55445
         ELSE
         ENDIF
C  DETERMINE IF A PHASE BOUNDARY HAS BEEN REACHED TO BEGIN ESTIMATE
C  COMPUTATION
       IF(I.EQ.1) GOTO 225
       IF(A(IPHASE,I).NE.A(IPHASE,(I-1))) THEN
C  COMPUTE THE WEIGHTED AVERAGE ESTIMATE
       MAX = 0
       K1 = K1 + 1
C  DETERMINE THE FAILURE CAUSE WITH THE LARGEST # OF FAILURES
       DO 220 I1=1,NCAUSE
          IF(CUMSF(1,I1).GT.MAX) THEN
          MAX = CUMSF(1,I1)
          ICOL = I1
          ELSE
          ENDIF
 220  CONTINUE
C COMPUTE YJK VALUE FOR THE CURRENT PHASE ESTIMATE
       IF(CUMSF(1,ICOL).LE.1000) THEN
          AHATL = YJK(CUMSF(1,ICOL))
       ELSEIF(CUMSF(1,ICOL).GT.1000) THEN
          X = CUMSF(1,ICOL)
          AHATL = (X**.10995)*3.55445
       ELSE
       ENDIF
       IX = CUMSF(1,ICOL) + CUMSF(3,ICOL)
       IF(IX.LE.1000) THEN
```

```
          AHATU = YJK(IX)
      ELSEIF(IX.GT.1000) THEN
          X = IX
          AHATU = (X**.10995)*3.55445
      ELSE
      ENDIF
C COMPUTE CURRENT PHASE RELIABILITY ESTIMATE
      AHAT = AHATU - AHATL
      CREL = 1.0 - EXP(-AHAT)
      X = CUMSF(1,ICOL) + CUMSF(3,ICOL)
C  COMPUTE AND STORE THE WEIGHTED AVERAGE ESTIMATE
      PREL = ((LTRIAL*PREL)/X) + (((X-LTRIAL)*CREL)/X)
      LTRIAL = CUMSF(1,ICOL) + CUMSF(3,ICOL)
C  COMPUTE THE PHASE AND GLOBAL AVERAGE FOR YJK USED IN THE WOODS
C  REGRESSION ESTIMATES ARE
      REG(2,K1) = TYJK/(I-1)
      REG(3,K1) = TPYJK/NFAPH(K1)
      TPYJK = 0.0
      ENDIF
C  COMPUTE THE NUMBER OF FAILURES AND SUCCESSES FOR EACH FAILURE CAUSE
C  USED IN THE WEIGHTED AVERAGE ESTIMATE
 225  ICAUSE = INT(A(IFAILC,I)+.5)
      DO 230 I1=1,NCAUSE
          CUMSF(2,I1) = CUMSF(2,I1) + INT(A(INTR,I) + .5)
          CUMSF(3,I1) = CUMSF(3,I1) + N1
 230  CONTINUE
      CUMSF(1,ICAUSE) = CUMSF(1,ICAUSE) + 1
      CUMSF(2,ICAUSE) = CUMSF(2,ICAUSE) - 1
      CUMSF(3,ICAUSE) = CUMSF(3,ICAUSE) - 1
           TPYJK = TPYJK + A(IYJK,I)
           TYJK = TYJK + A(IYJK,I)
 240   CONTINUE
C  REPEAT COMPUTATIONS FOR THE WEIGHTED AVERAGE ESTIMATE FOR THE
C  FINAL PHASE
      MAX = 0
      K1 = K1 + 1
      DO 245 I1=1,NCAUSE
          IF(CUMSF(1,I1).GT.MAX) THEN
          MAX = CUMSF(1,I1)
          ICOL = I1
          ELSE
          ENDIF
 245  CONTINUE
      IF(CUMSF(1,ICOL).LE.1000) THEN
          AHATL = YJK(CUMSF(1,ICOL))
      ELSEIF(CUMSF(1,ICOL).GT.1000) THEN
          X = CUMSF(1,ICOL)
          AHATL = (X**.10995)*3.55445
      ELSE
      ENDIF
      IX = CUMSF(1,ICOL) + CUMSF(3,ICOL)
      IF(IX.LE.1000) THEN
          AHATU = YJK(IX)
      ELSEIF(IX.GT.1000) THEN
          X = IX
          AHATU = (X**.10995)*3.55445
      ELSE
      ENDIF
      AHAT = AHATU - AHATL
      CREL = 1.0 - EXP(-AHAT)
      X = CUMSF(1,ICOL) + CUMSF(3,ICOL)

      PREL = ((LTRIAL*PREL)/X) + (((X-LTRIAL)*CREL)/X)
```

```
      LTRIAL = CUMSF(1,ICOL) + CUMSF(3,ICOL)
      REG(2,K1) = TYJK/(J)
      REG(3,K1) = TPYJK/NFAPH(K1)
        PHREST(1,K) = PREL
C  COMPUTE THE MLE ESTIMATE OF PHASE RELIABILITY USING DISCOUNTING
        PHREST(2,K) = (TADJT - J)/TADJT
C  COMPUTE THE EXPONENTIAL REGRESSION ESTIMATE BEGINNING WITH B HAT
         SUM = 0.0
         SUMS = 0.0
         IF (K.EQ.1) GOTO 252
         DO 250 I = 1,K
               SUM = SUM + ((I-REG(1,K))*REG(3,I))
               SUMS = SUMS + ((I-REG(1,K))**2)
 250    CONTINUE
         REG(4,K) = SUM/SUMS
C  COMPUTE A HAT
         REG(5,K) = REG(2,K) - (REG(4,K)*REG(1,K))
C  COMPUTE AND STORE THE EXPONENTIAL REGRESSION ESTIMATE
         PHREST(4,K) = 1.0 - EXP(-(REG(5,K) + (REG(4,K)*K)))
         IF(PHREST(4,K).LT.0.0) PHREST(4,K)=0.0
         GOTO 255
 252    PHREST(4,K) = 1.0 - EXP(-REG(3,1))
         IF(PHREST(4,K).LT.0.0) PHREST(4,K)=0.0
C  STORE THE RUNNING SUM OF THE ESTIMATES FOR THE CURRENT PHASE AND THE
C  RUNNING SUM OF THE ESTIMATES SQUARED FOR COMPUTATION OF THE MEAN AND
C  STANDARD DEVIATION OF EACH ESTIMATE FOR EACH RELIABILITY GROWTH
C  MODEL
 255   EST(1,K) = EST(1,K) + PHREST(1,K)
        EST(2,K) = EST(2,K) + (PHREST(1,K)**2)
        MLEWD(1,K) = MLEWD(1,K) + PHREST(2,K)
        MLEWD(2,K) = MLEWD(2,K) + (PHREST(2,K)**2)
        REGEST(1,K) = REGEST(1,K) + PHREST(4,K)
        REGEST(2,K) = REGEST(2,K) + (PHREST(4,K)**2)
C  STORE THE ACTUAL PHASE RELIABILITY
        AREL(K) = A(ISYSPR,J)
C  PRINT INTERMEDIATE OUTPUT IF REQUESTED AND THE NUMBER OF REPETITIONS
C  IS NOT GREATER THAN 5
        IF(IOPT.NE.1) GOTO 300
        IF(REP.GT.5) GOTO 300
      WRITE(20,1000) REP,K
 1000 FORMAT(T16,'REPETITION NUMBER: ',I4,'   PHASE NUMBER: ',I4)
      WRITE(20,1010) A(ISYSPR,J)
 1010 FORMAT(22X,'ACTUAL COMPONENT RELIABILITY: ',F7.5)
      WRITE(20,1020) PHREST(1,K)
 1020 FORMAT(20X,'PREDICTED COMPONENT RELIABILITY: ',F7.5)
      WRITE(20,1022) PHREST(2,K)
 1022 FORMAT(20X,'MLE ESTIMATE USING DISCOUNTING: ',F7.5)
      WRITE(20,1025) PHREST(3,K)
 1025 FORMAT(18X,'MLE ESTIMATE OF PHASE RELIABILITY: ',F7.5)
      WRITE(20,1027) PHREST(4,K)
 1027 FORMAT(14X,'REGRESSION ESTIMATE OF PHASE RELIABILITY: ',F7.5)
      WRITE(20,1030)
 1030 FORMAT(' ','      ')
      DO 260 I=1,NCAUSE
        WRITE(20,1035)I,A(I,J),A((I+NCAUSE),J)
 1035  FORMAT(12X,'CAUSE: ',I3,'   PR(SUCCESS): ',F7.6,'   # TRIALS: ',
     CF10.0)
 260  CONTINUE
      WRITE(20,1036)
 1036 FORMAT(' ','      ')
      WRITE(20,1040)
```

```
 1040 FORMAT(4X,'FAIL #',3X,'FAIL CAUSE',3X,'# TRIALS',3X,'ADJ # FAIL',3
     CX,'ADJ # TRIALS',7X,'YJK')
      DO 270 I=1,J
      WRITE(20,1050)I,A(IFAILC,I),A(INTR,I),A(IADJF,I),A(IADJT,I),A(IYJK
     C,I)
 1050 FORMAT(4X,I3,8X,F3.0,7X,F8.0,4X,F8.6,4X,F12.0,3X,F11.4)
 270  CONTINUE
      WRITE(20,1060)
 1060 FORMAT(' ',///)
 300  CONTINUE
C  PRINT EACH OF THE 3 ESTIMATES TO THEIR APPROPRIATE OUTPUT FILE
C  IF REQUESTED
      IF(IOPT1.NE.1) GOTO 401
 400  WRITE(40,2000) (PHREST(1,I), I=1,NPHASE)
 401  IF(IOPT2.NE.1) GOTO 402
      WRITE(50,2000) (PHREST(2,I), I=1,NPHASE)
 402  IF(IOPT3.NE.1) GOTO 403
      WRITE(60,2000) (PHREST(3,I), I=1,NPHASE)
 403  IF(IOPT4.NE.1) GOTO 500
      WRITE(70,2000) (PHREST(4,I), I=1,NPHASE)
 2000 FORMAT(' ',30(F7.6:1X))
 500  CONTINUE
C  UPON COMPLETION OF ALL REPETITIONS, COMPUTE THE MEAN AND STANDARD
C  DEVIATION OF EACH ESTIMATE FOR EACH PHASE SKIPPING COMPUTATIONS IF
C  ONLY ONE REPETITION IS REQUIRED
      IF (NREP.LE.1) GOTO 601
      DO 600 I=1,NPHASE
      EST(3,I) = EST(1,I)/XNREP
      MLEWD(3,I) = MLEWD(1,I)/XNREP
      MLESP(3,I) = MLESP(1,I)/XNREP
      REGEST(3,I) = REGEST(1,I)/XNREP
      EST(4,I) = SQRT((EST(2,I)-(XNREP*(EST(3,I)**2)))/(XNREP-1))
      MLEWD(4,I) = SQRT((MLEWD(2,I)-(XNREP*(MLEWD(3,I)**2)))/(XNREP-1))
      MLESP(4,I) = SQRT((MLESP(2,I)-(XNREP*(MLESP(3,I)**2)))/(XNREP-1))
      REGEST(4,I)=SQRT((REGEST(2,I)-(XNREP*(REGEST(3,I)**2)))/(XNREP-1))
 600  CONTINUE
C  PRINT THE FINAL OUTPUT TABLE TO A FILE
 601  WRITE(30,3000)
 3000 FORMAT('0',T47,'DISCRETE RELIABILITY GROWTH SIMULATION')
      WRITE(30,3010)
 3010 FORMAT('-',T54,'MODEL PARAMETER SUMMARY')
      WRITE(30,3020) NCAUSE
 3020 FORMAT('0',T47,'NUMBER OF POSSIBLE FAILURE CAUSES ',I4)
      WRITE(30,3030)
 3030 FORMAT('0',T38,'CAUSE NUMBER',T64,'SINGLE TRIAL PR( SUCCESS ) FOR
     CPHASE 1')
      DO 3050 M=1,NCAUSE
      WRITE(30,3040) M,A(M,1)
 3040 FORMAT(' ',T43,I2,T79,F8.6)
 3050 CONTINUE
      WRITE(30,3060) FRIMP
 3060 FORMAT('0',T37,'FRACTION CAUSE RELIABILITY IMPROVES AFTER FAILURE
     C',F8.6)
      WRITE(30,3080) NPHASE
 3080 FORMAT('-',T48,'NUMBER OF PHASES IN THE SIMULATION ',I2)
      WRITE(30,3090)
 3090 FORMAT('0',T42,'PHASE NUMBER',T59,'NUMBER OF FAILURES IN THE FIRST
     C PHASE')
      DO 3110 M=1,NPHASE
      WRITE(30,3100) M,NFAPH(M)
 3100 FORMAT(' ',T43,I2,T73,I2)
 3110 CONTINUE
      WRITE(30,3120) NFAIL
 3120 FORMAT('0',T51,'TOTAL NUMBER OF FAILURES ',I4)
      IF(DISOPT.EQ.2) GO TO 3160
      WRITE(30,3130)
```

```
 3130 FORMAT('-',T38,'DISCOUNTING PERFORMED USING THE CONSTANT FRACTION
     CMETHOD')
      WRITE(30,3140) R
 3140 FORMAT('0',T44,'FRACTION EACH FAILURE IS DISCOUNTED ',F8.6)
      WRITE(30,3150) N
 3150 FORMAT(' ',T33,'NUMBER OF TRIALS AFTER A FAILURE BEFORE A DISCOUNT
     C IS APPLIED ',I4)
      GO TO 3190
 3160 WRITE(30,3170)
 3170 FORMAT('-',T44,'DISCOUNTING PERFORMED USING THE LLOYD METHOD')
      WRITE(30,3180) GAMA
 3180 FORMAT('0',T39,'PERCENT C.I. ( USED AS DISCOUNT FRACTION ) ',F8.6
     C)
 3190 WRITE(30,3200) DSEED1
 3200 FORMAT('-',T46,'RANDOM NUMBER SEED USED ',F15.2)
      WRITE(30,3210) NREP
 3210 FORMAT('0',T37,'NUMBER OF REPETITIONS OF THE SIMULATION PERFORMED
     C',I7)
      WRITE(30,3220)
 3220 FORMAT('1',T61,'ESTIMATOR:')
      WRITE(30,3230)
 3230 FORMAT('0',T48,'SINGLE PHASE MLE WITHOUT DISCOUNTING')
      WRITE(30,3240)
 3240 FORMAT('-',T60,'MEAN',T83,'ESTIMATE',T109,'95 %')
      WRITE(30,3250)
 3250 FORMAT(' ',T12,'PHASE NUMBER',T29,'ACTUAL RELIABILITY',T52,'PREDIC
     CTED RELIABILITY',T78,'STANDARD DEVIATION',T101,'CONFIDENCE INTERVA
     CL')
C  COMPUTE C.I. FOR SINGLE PHASE MLE
      DO 3270 M=1,NPHASE
      CI = (1.96*MLESP(4,M))/SQRT(XNREP)
      CIU = MLESP(3,M) + CI
      CIL = MLESP(3,M) - CI
      WRITE(30,3260) M,AREL(M),MLESP(3,M),MLESP(4,M),CIL,CIU
 3260 FORMAT('0',T17,I2,T34,F8.6,T58,F8.6,T82,F9.6,T99,'( ',F8.6,' , ',F
     C8.6,' )')
 3270 CONTINUE
      WRITE(30,3220)
      WRITE(30,3280)
 3280 FORMAT('0',T42,'MAX LIKELIHOOD ESTIMATE USING DISCOUNTED FAILURES'
     C)
      WRITE(30,3240)
      WRITE(30,3250)
C  COMPUTE C.I. FOR MLE WITH DISCOUNTING
      DO 3290 M=1,NPHASE
      CI = (1.96*MLEWD(4,M))/SQRT(XNREP)
      CIU = MLEWD(3,M) + CI
      CIL = MLEWD(3,M) - CI
      WRITE(30,3260) M,AREL(M),MLEWD(3,M),MLEWD(4,M),CIL,CIU
 3290 CONTINUE
      WRITE(30,3220)
      WRITE(30,3300)
 3300 FORMAT('0',T38,'WEIGHTED AVERAGE ESTIMATE USING FAILURE DISCOUNTIN
     CG')
      WRITE(30,3240)
      WRITE(30,3250)
C  COMPUTE C.I. FOR WEIGHTED AVERAGE ESTIMATES
      DO 3310 M=1,NPHASE
      CI = (1.96*EST(4,M))/SQRT(XNREP)
      CIU = EST(3,M) + CI
      CIL = EST(3,M) - CI
      WRITE(30,3260) M,AREL(M),EST(3,M),EST(4,M),CIL,CIU
 3310 CONTINUE
      WRITE(30,3220)
      WRITE(30,3320)
 3320 FORMAT('0',T43,'REGRESSION ESTIMATE USING DISCOUNTED FAILURES')
```

```
      WRITE(30,3240)
      WRITE(30,3250)
C  COMPUTE C.I. FOR EXPONENTIAL REGRESSION ESTIMATES
      DO 3330 M=1,NPHASE
      CI = (1.96*REGEST(4,M))/SQRT(XNREP)
      CIU = REGEST(3,M) + CI
      CIL = REGEST(3,M) - CI
      WRITE(30,3260) M,AREL(M),REGEST(3,M),REGEST(4,M),CIL,CIU
 3330 CONTINUE
      WRITE(30,3340)
 3340 FORMAT('1',T59,'RECAPITULATION'//)
      WRITE(30,3350)
 3350 FORMAT('-',T3,'PHASE',T11,'ACTUAL',T28,'MEAN',T38,'EST',T53,'MEAN'
     C,T63,'EST',T78,'MEAN',T88,'EST',T103,'MEAN',T113,'EST')
      WRITE(30,3360)
 3360 FORMAT(' ',T11,'RELIAB',T28,'WGT',T38,'STD',T53,'MLE',T63,'STD',T7
     C7,'PHASE',T88,'STD',T103,'REG',T113,'STD')
      WRITE(30,3370)
 3370 FORMAT(' ',T28,'AVG',T35,'DEVIATION',T53,'W/D',T60,'DEVIATION',T78
     C,'MLE',T85,'DEVIATION',T103,'EST',T110,'DEVIATION')
      WRITE(30,3375)
 3375 FORMAT(' ',T28,'EST'/)
      DO 650 I=1,NPHASE
         WRITE(30,3380)I,AREL(I),EST(3,I),EST(4,I),MLEWD(3,I),MLEWD(4,I),
     CMLESP(3,I),MLESP(4,I),REGEST(3,I),REGEST(4,I)
 3380 FORMAT('0',T4,I3,T11,F7.6,T26,F7.6,T36,F7.6,T51,F7.6,T61,F7.6,T76,
     CF7.6,T86,F7.6,T101,F7.6,T111,F7.6)
 650  CONTINUE

      STOP
      END
```

2. SAMPLE SIMULATION INPUT FILE

5	NUMBER OF FAILURE CAUSES
10	NUMBER OF PHASES (NPHASE)
1	NUMBER OF FAILURES IN PHASE 1
1	NUMBER OF FAILURES IN PHASE 2
1	NUMBER OF FAILURES IN PHASE 3
1	NUMBER OF FAILURES IN PHASE 4
1	NUMBER OF FAILURES IN PHASE 5
1	NUMBER OF FAILURES IN PHASE 6
1	NUMBER OF FAILURES IN PHASE 7
1	NUMBER OF FAILURES IN PHASE 8
1	NUMBER OF FAILURES IN PHASE 9
1	NUMBER OF FAILURES IN PHASE 10
.8	1 - PROB. OF OCCURRENCE FOR CAUSE 1 IN PHASE 1
.8	1 - PROB. OF OCCURRENCE FOR CAUSE 2 IN PHASE 1
.8	1 - PROB. OF OCCURRENCE FOR CAUSE 3 IN PHASE 1
.7	1 - PROB. OF OCCURRENCE FOR CAUSE 4 IN PHASE 1
.7	1 - PROB. OF OCCURRENCE FOR CAUSE 5 IN PHASE 1
50	DISCOUNT INTERVAL FOR STRAIGHT PERCENT DISCOUNTING
.75	DISCOUNT FRACTION FOR STRAIGHT PERCENT DISCOUNTING
624712.0	RANDOM NUMBER SEED
.95	RELIABILITY GROWTH FRACTION
10000	NUMBER OF DESIRED REPETITIONS FOR THE SIMULATION
0	INTERMEDIATE OUTPUT OPTION (1:INT. OUTPUT; 0: NO OUTPUT)
0	SAVE ALL WEIGHTED AVERAGE ESTIMATES (1: YES; 0: NO)
0	SAVE ALL MLE W/ DISCOUNTING ESTIMATES (1: YES; 0: NO)
0	SAVE ALL MLE SINGLE PHASE ESTIMATES (1: YES; 0: NO)
0	SAVE ALL REGRESSION ESTIMATES (1: YES; 0: NO)
2	DISCOUNTING OPTION (1: STRAIGHT % ; 2: LLOYD METHOD)
.2	GAMMA FOR LLOYD DISCOUNTING METHOD

3. SAMPLE SIMULATION OUTPUT FILE

```
                DISCRETE RELIABILITY GROWTH SIMULATION

                        MODEL PARAMETER SUMMARY

                NUMBER OF POSSIBLE FAILURE CAUSES     5

   CAUSE NUMBER                 SINGLE TRIAL PR( SUCCESS ) FOR PHASE 1
        1                                  0.800000
        2                                  0.800000
        3                                  0.800000
        4                                  0.700000
        5                                  0.700000

  FRACTION CAUSE RELIABILITY IMPROVES AFTER FAILURE 0.950000

                NUMBER OF PHASES IN THE SIMULATION 10

       PHASE NUMBER     NUMBER OF FAILURES IN THE FIRST PHASE
        1                          1
        2                          1
        3                          1
        4                          1
        5                          1
        6                          1
        7                          1
        8                          1
        9                          1
       10                          1

                  TOTAL NUMBER OF FAILURES   10

      DISCOUNTING PERFORMED USING THE CONSTANT FRACTION METHOD

             FRACTION EACH FAILURE IS DISCOUNTED 0.750000
 NUMBER OF TRIALS AFTER A FAILURE BEFORE A DISCOUNT IS APPLIED   50

               RANDOM NUMBER SEED USED       624712.00

   NUMBER OF REPETITIONS OF THE SIMULATION PERFORMED      100
```

ESTIMATOR:

SINGLE PHASE MLE WITHOUT DISCOUNTING

PHASE NUMBER	ACTUAL RELIABILITY	MEAN PREDICTED RELIABILITY	ESTIMATE STANDARD DEVIATION	95 % CONFIDENCE INTERVAL
1	0.250880	0.155833	0.248355	(0.107156 , 0.204511)
2	0.310464	0.149167	0.251063	(0.099958 , 0.198375)
3	0.436867	0.227500	0.288640	(0.170927 , 0.284073)
4	0.614734	0.454739	0.339315	(0.388233 , 0.521244)
5	0.760734	0.543383	0.317032	(0.481245 , 0.605521)
6	0.941408	0.808771	0.261342	(0.757547 , 0.859994)
7	0.955027	0.864222	0.196500	(0.825708 , 0.902736)
8	0.964192	0.904001	0.160813	(0.872481 , 0.935520)
9	0.973444	0.943251	0.078030	(0.927957 , 0.958544)
10	0.987527	0.949338	0.127139	(0.924419 , 0.974257)

ESTIMATOR:

MAX LIKELIHOOD ESTIMATE USING DISCOUNTED FAILURES

PHASE NUMBER	ACTUAL RELIABILITY	MEAN PREDICTED RELIABILITY	ESTIMATE STANDARD DEVIATION	95 % CONFIDENCE INTERVAL
1	0.250880	0.155833	0.248355	(0.107156 , 0.204511)
2	0.310464	0.193286	0.216518	(0.150848 , 0.235723)
3	0.436867	0.242643	0.208690	(0.201739 , 0.283546)
4	0.614734	0.385568	0.203544	(0.345674 , 0.425463)
5	0.760734	0.489647	0.173475	(0.455646 , 0.523648)
6	0.941408	0.695538	0.163022	(0.663586 , 0.727490)
7	0.955027	0.818813	0.108281	(0.797590 , 0.840036)
8	0.964192	0.894025	0.076595	(0.879012 , 0.909037)
9	0.973444	0.944770	0.057664	(0.933468 , 0.956073)
10	0.987527	0.981208	0.032805	(0.974778 , 0.987638)

ESTIMATOR:

REGRESSION ESTIMATE USING DISCOUNTED FAILURES

PHASE NUMBER	ACTUAL RELIABILITY	MEAN PREDICTED RELIABILITY	ESTIMATE STANDARD DEVIATION	95 % CONFIDENCE INTERVAL
1	0.250880	0.192632	0.304892	(0.132873 , 0.252391)
2	0.310464	0.182702	0.304595	(0.123001 , 0.242403)
3	0.436867	0.295146	0.309750	(0.234435 , 0.355857)
4	0.614734	0.518837	0.302365	(0.459573 , 0.578101)
5	0.760734	0.662013	0.231360	(0.616667 , 0.707360)
6	0.941408	0.841218	0.173684	(0.807176 , 0.875260)
7	0.955027	0.925839	0.063032	(0.913485 , 0.938193)
8	0.964192	0.960515	0.037333	(0.953197 , 0.967832)
9	0.973444	0.979089	0.022257	(0.974727 , 0.983452)
10	0.987527	0.987879	0.017359	(0.984477 , 0.991282)

ESTIMATOR:

WEIGHTED AVERAGE ESTIMATE USING FAILURE DISCOUNTING .

PHASE NUMBER	ACTUAL RELIABILITY	MEAN PREDICTED RELIABILITY	ESTIMATE STANDARD DEVIATION	95 % CONFIDENCE INTERVAL
1	0.250880	0.192632	0.304892	(0.132873 , 0.252391)
2	0.310464	0.399806	0.239711	(0.352822 , 0.446789)
3	0.436867	0.496501	0.215567	(0.454250 , 0.538752)
4	0.614734	0.607152	0.171114	(0.573613 , 0.640690)
5	0.760734	0.677896	0.129972	(0.652421 , 0.703370)
6	0.941408	0.793632	0.102869	(0.773469 , 0.813794)
7	0.955027	0.874020	0.078053	(0.858721 , 0.889318)
8	0.964192	0.927141	0.060652	(0.915253 , 0.939029)
9	0.973444	0.963650	0.044136	(0.954999 , 0.972301)
10	0.987527	0.988730	0.023216	(0.984179 , 0.993280)

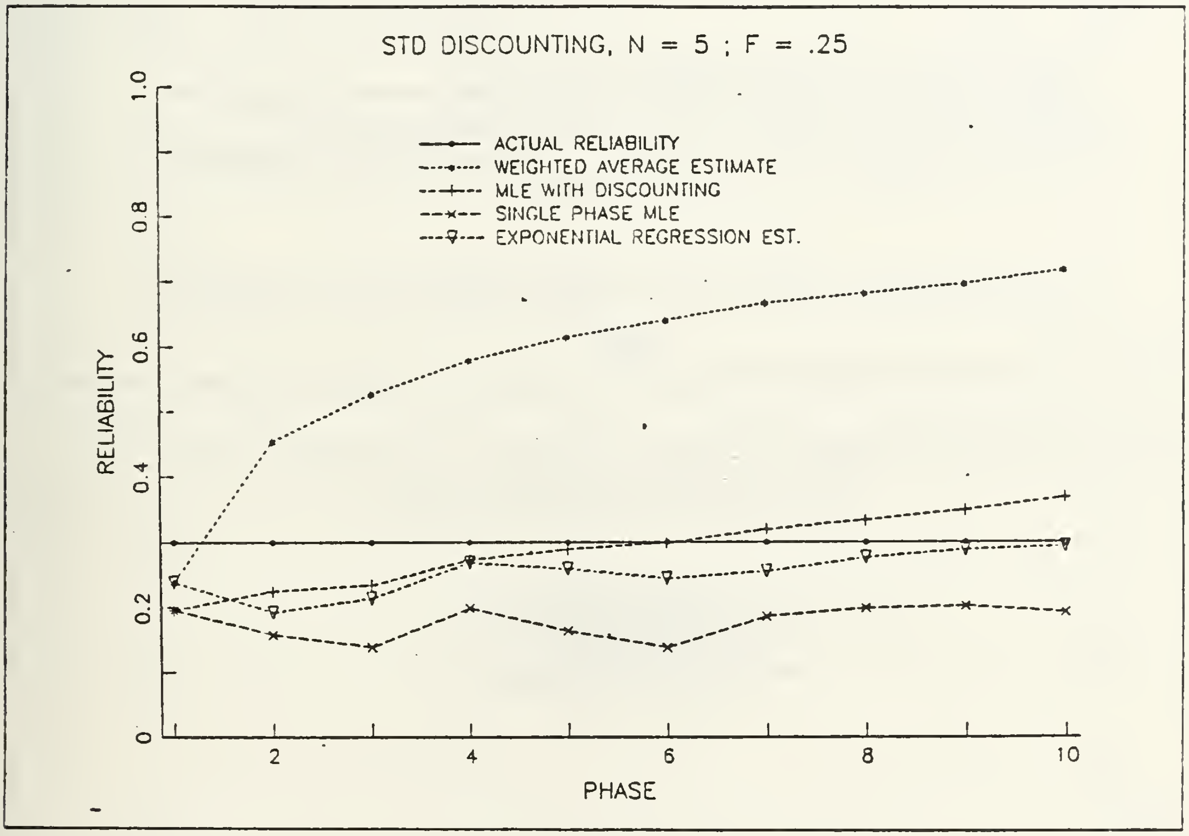
RECAPITULATION

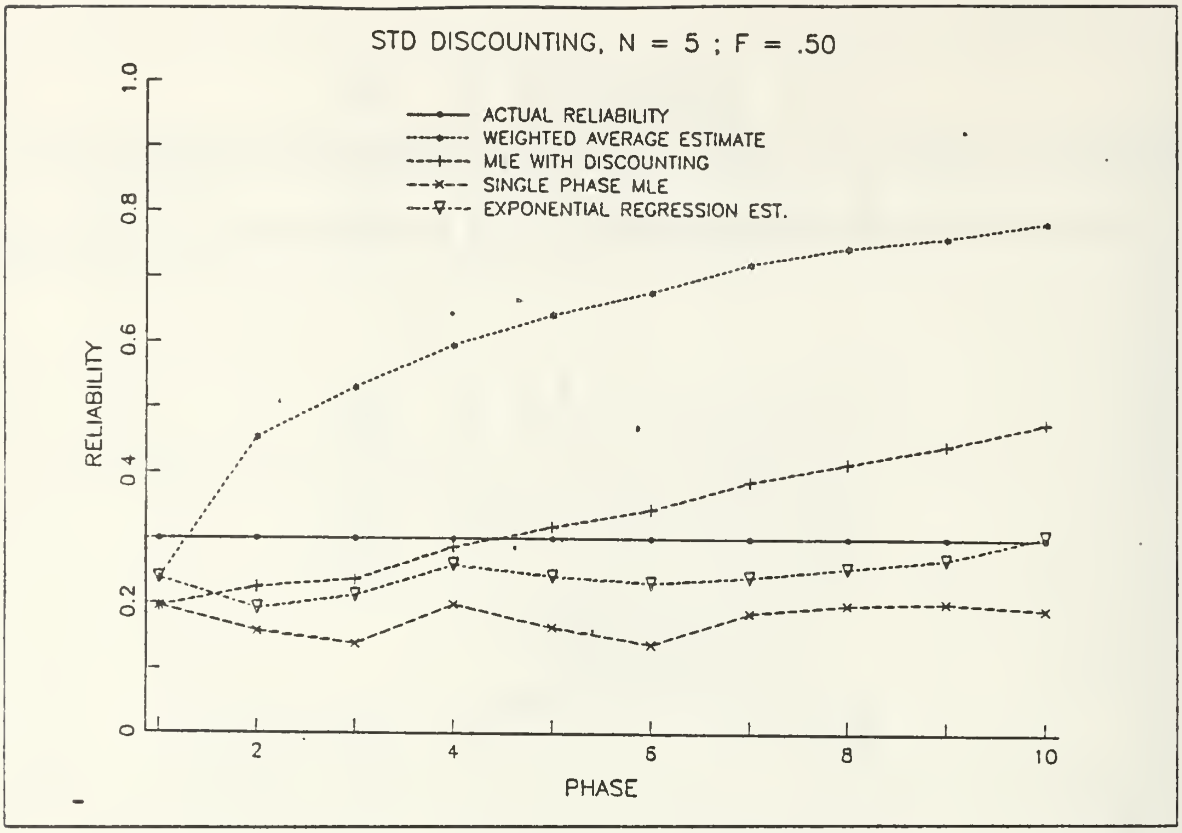
PHASE	ACTUAL RELIAB	MEAN WGT AVG EST	EST STD DEVIATION	MEAN MLE W/D	EST STD DEVIATION	MEAN PHASE MLE	EST STD DEVIATION	MEAN REG EST	EST STD DEVIATION
1	.250880	.192632	.304892	.155833	.248355	.155833	.248355	.192632	.304892
2	.310464	.399806	.239711	.193286	.216518	.149167	.251063	.182702	.304595
3	.436867	.496501	.215567	.242643	.208690	.227500	.288640	.295146	.309750
4	.614734	.607152	.171114	.385568	.203544	.454739	.339315	.518837	.302365
5	.760734	.677896	.129972	.489647	.173475	.543383	.317032	.662013	.231360
6	.941408	.793632	.102869	.695538	.163022	.808771	.261342	.841218	.173684
7	.955027	.874020	.078053	.818813	.108281	.864222	.196500	.925839	.063032
8	.964192	.927141	.060652	.894025	.076595	.904001	.160813	.960515	.037333
9	.973444	.963650	.044136	.944770	.057664	.943251	.078030	.979089	.022257
10	.987527	.988730	.023216	.981208	.032805	.949338	.127139	.987879	.017359

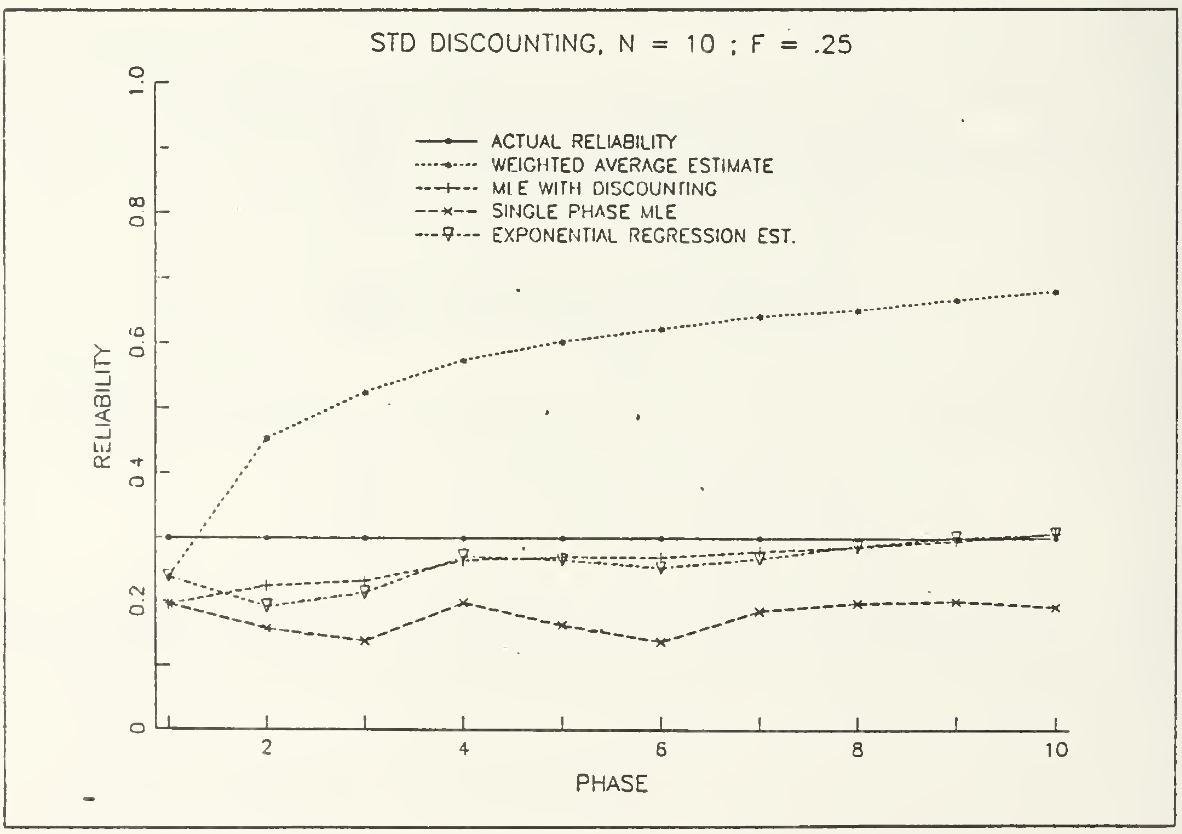
APPENDIX B

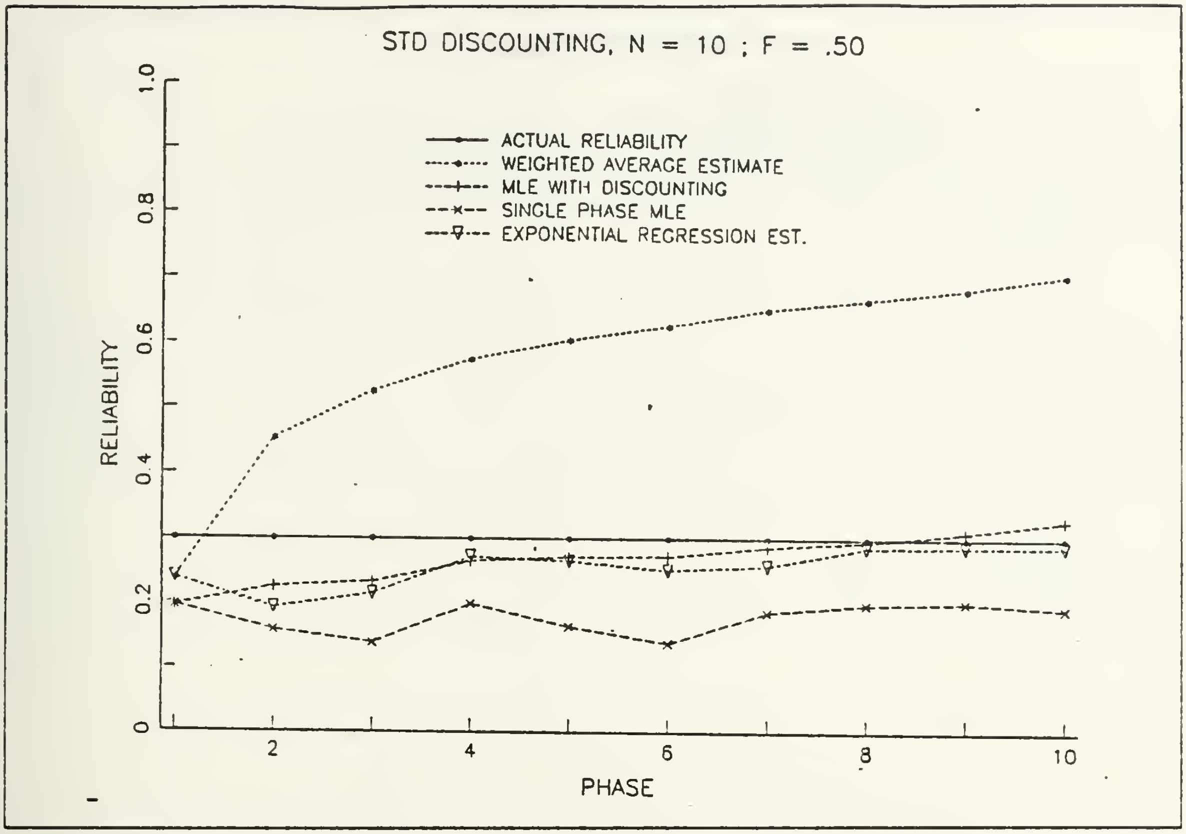
GRAPHICAL RESULTS OF THE RELIABILITY GROWTH MODEL SIMULATION

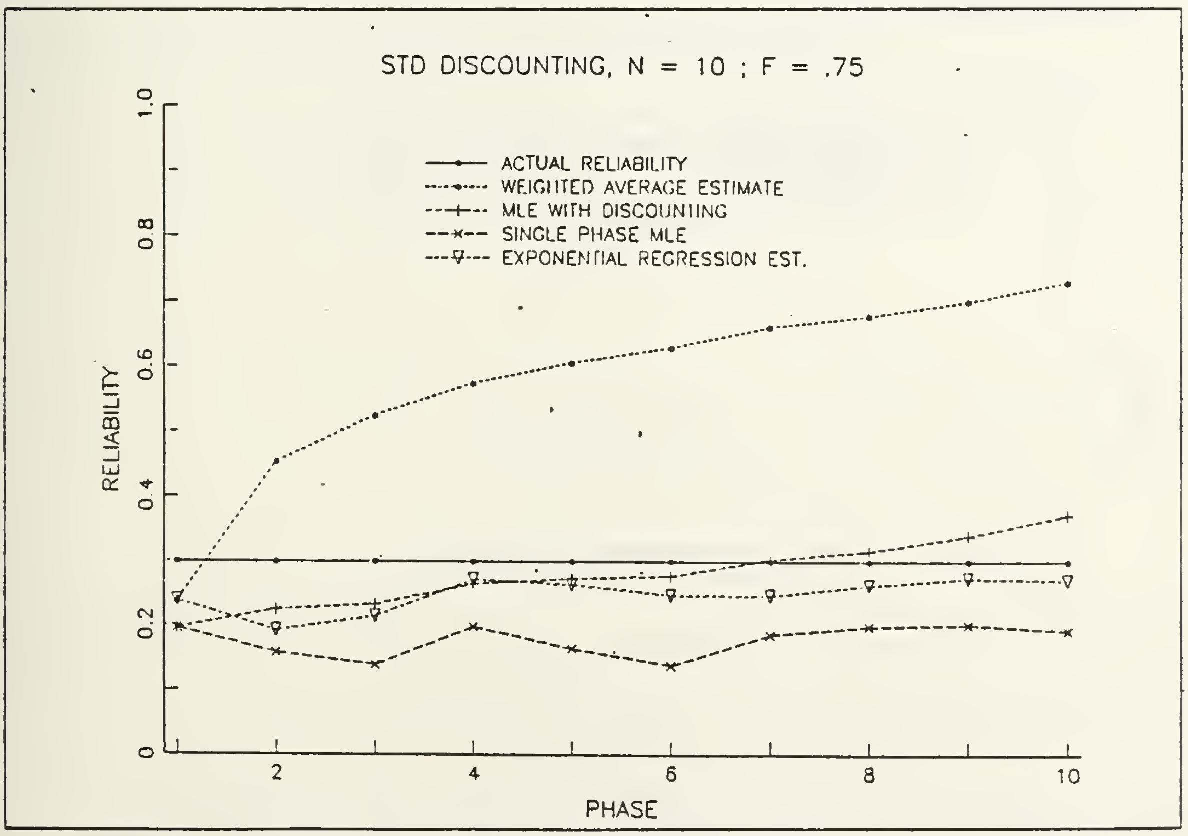
1. MEAN ESTIMATES FOR EACH ESTIMATOR WITH CONSTANT PARAMETERS

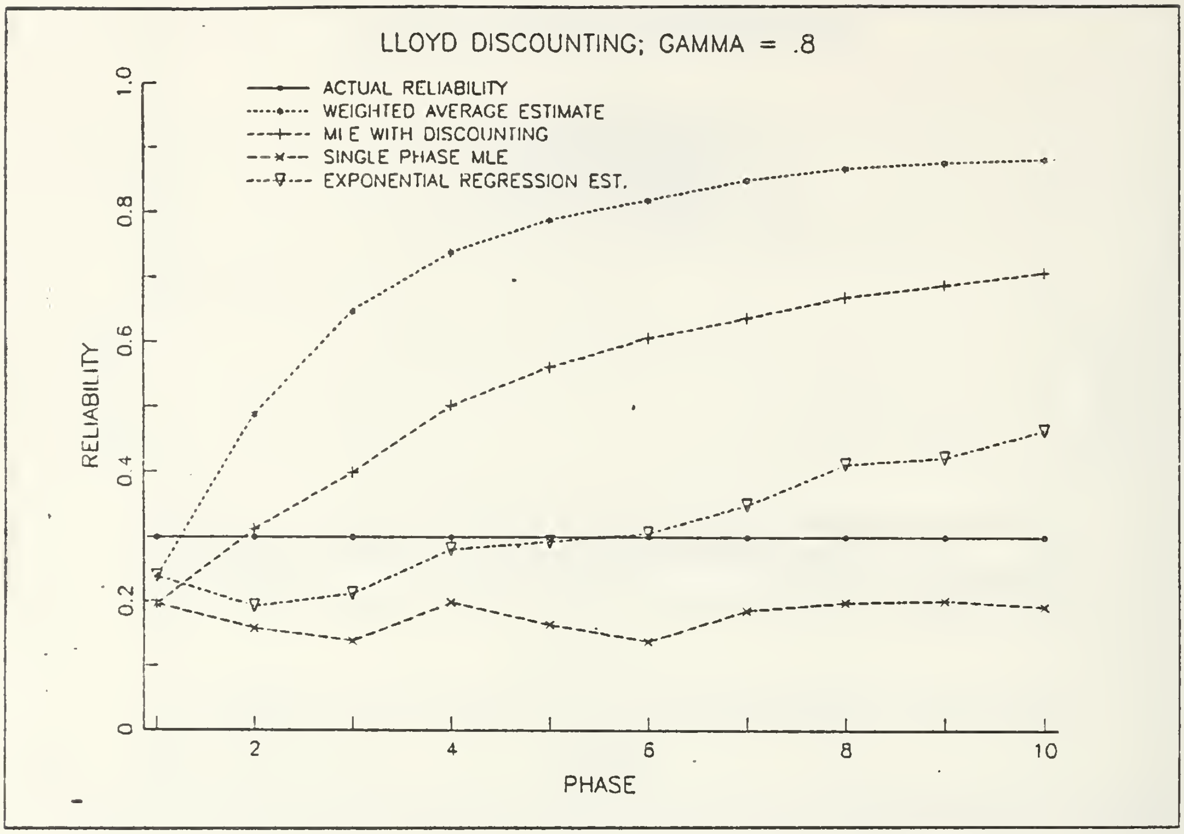



STD DISCOUNTING, N = 5 ; F = .50
ACTUAL RELIABILITY
WEIGHTED AVERAGE ESTIMATE
MLE WITH DISCOUNTING
SINGLE PHASE MLE
EXPONENTIAL REGRESSION EST.
RELIABILITY
0
0.2
0.4
0.6
0.8
1.0
2
4
6
8
10
PHASE

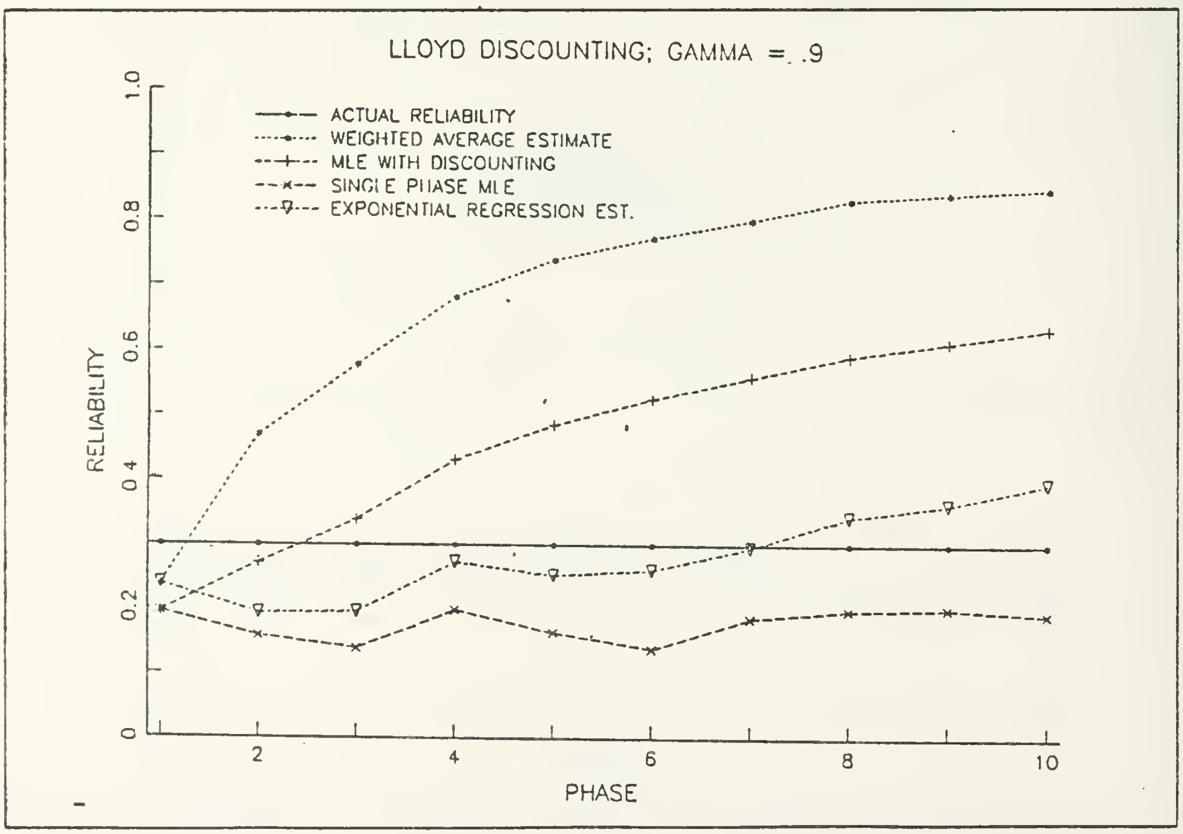

STD DISCOUNTING, N = 10 ; F = .25
ACTUAL RELIABILITY
WEIGHTED AVERAGE ESTIMATE
MLE WITH DISCOUNTING
SINGLE PHASE MLE
EXPONENTIAL REGRESSION EST.
RELIABILITY
0
0.2
0.4
0.6
0.8
1.0
2
4
6
8
10
PHASE


STD DISCOUNTING, N = 10 ; F = .50
ACTUAL RELIABILITY
WEIGHTED AVERAGE ESTIMATE
MLE WITH DISCOUNTING
SINGLE PHASE MLE
EXPONENTIAL REGRESSION EST.
RELIABILITY
0
0.2
0.4
0.6
0.8
1.0
2
4
6
8
10
PHASE

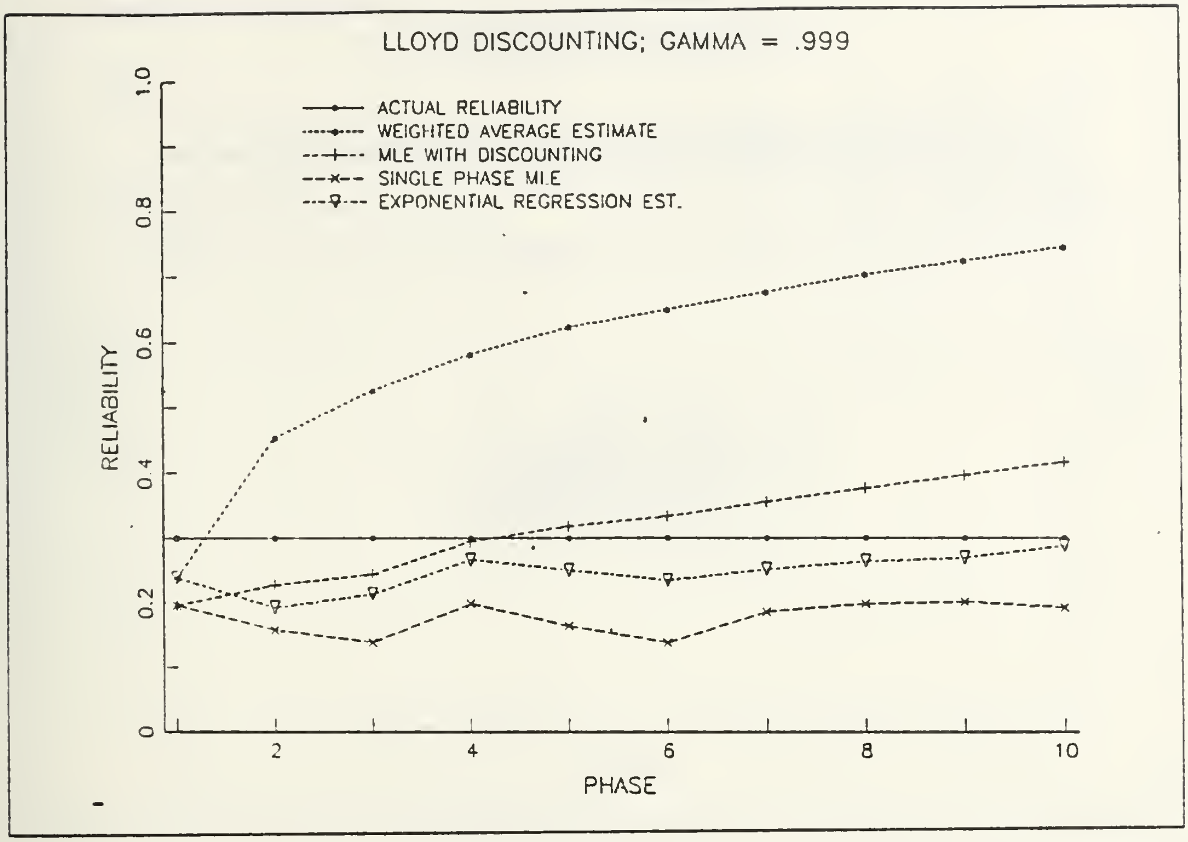

STD DISCOUNTING, N = 10 ; F = .75
ACTUAL RELIABILITY
WEIGHTED AVERAGE ESTIMATE
MLE WITH DISCOUNTING
SINGLE PHASE MLE
EXPONENTIAL REGRESSION EST.
RELIABILITY
0
0.2
0.4
0.6
0.8
1.0
2
4
6
8
10
PHASE

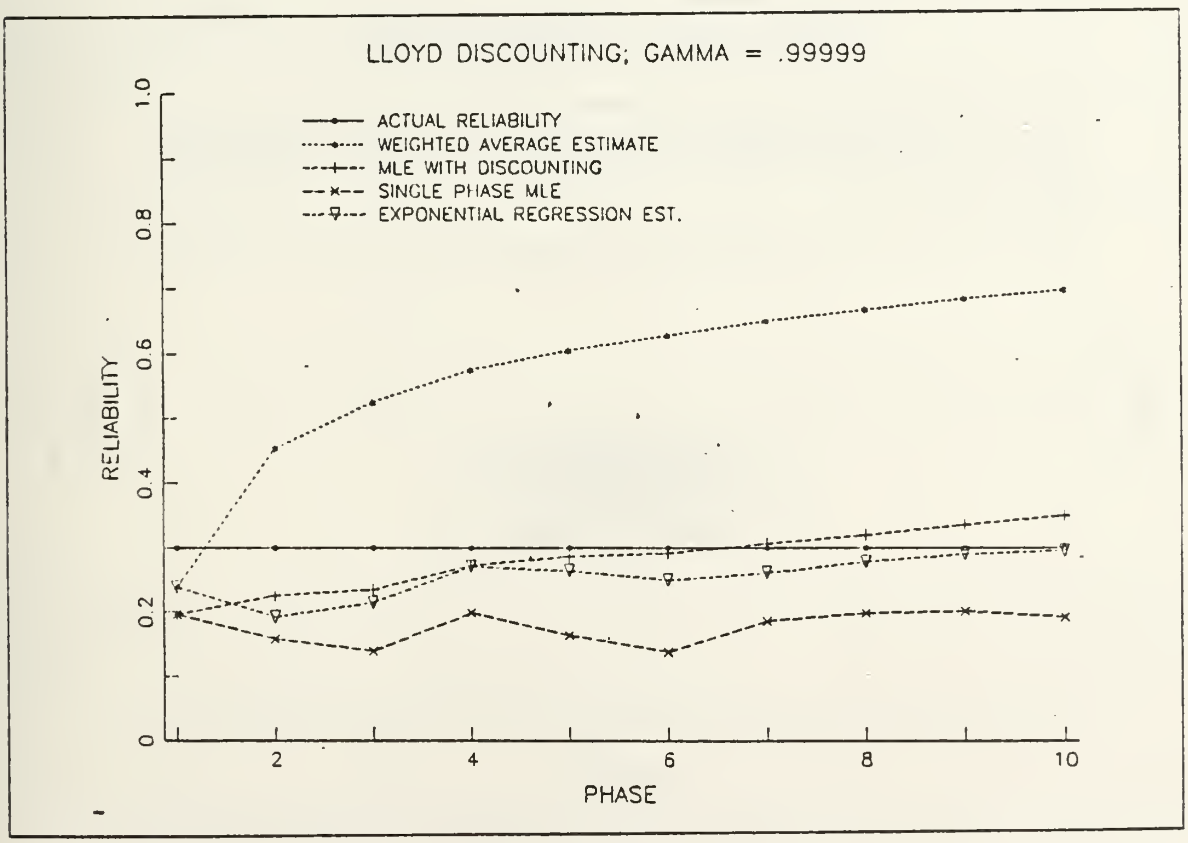


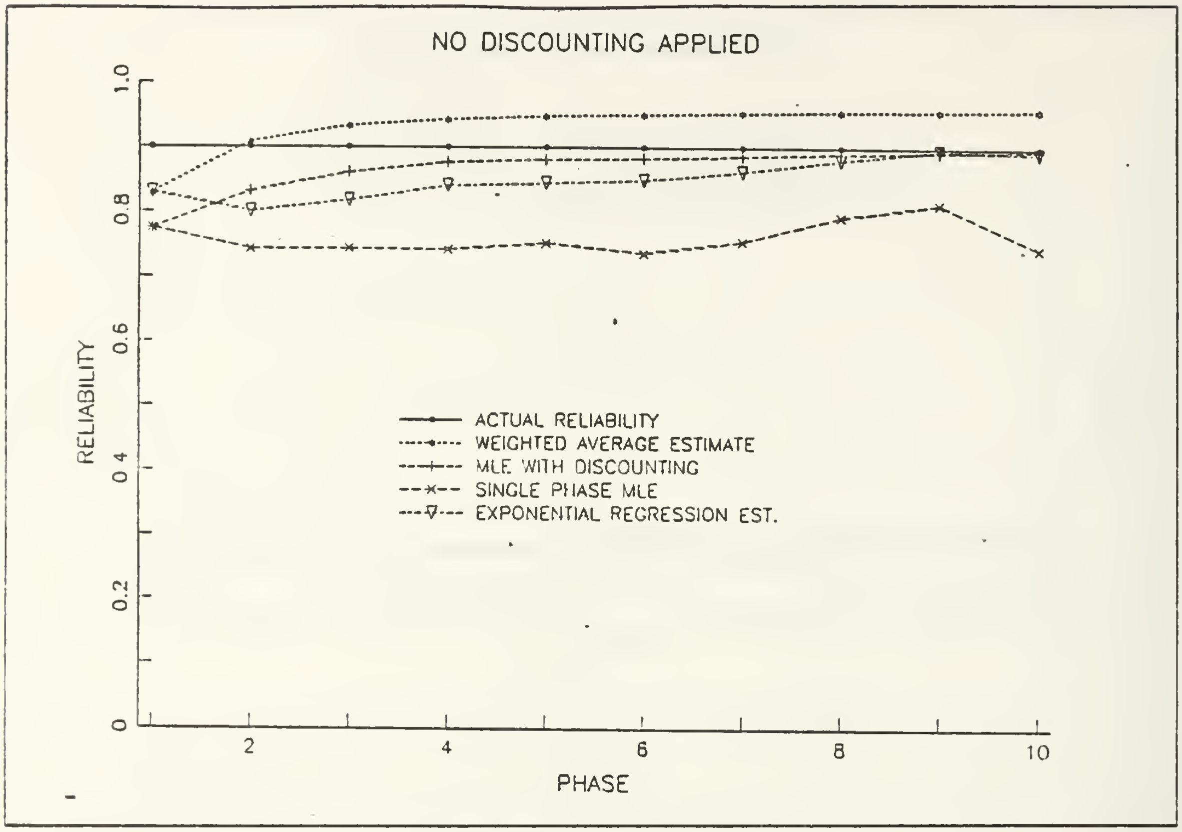
LLOYD DISCOUNTING; GAMMA = .8
ACTUAL RELIABILITY
WEIGHTED AVERAGE ESTIMATE
MLE WITH DISCOUNTING
SINGLE PHASE MLE
EXPONENTIAL REGRESSION EST.
RELIABILITY
0
0.2
0.4
0.6
0.8
1.0
2
4
6
8
10
PHASE

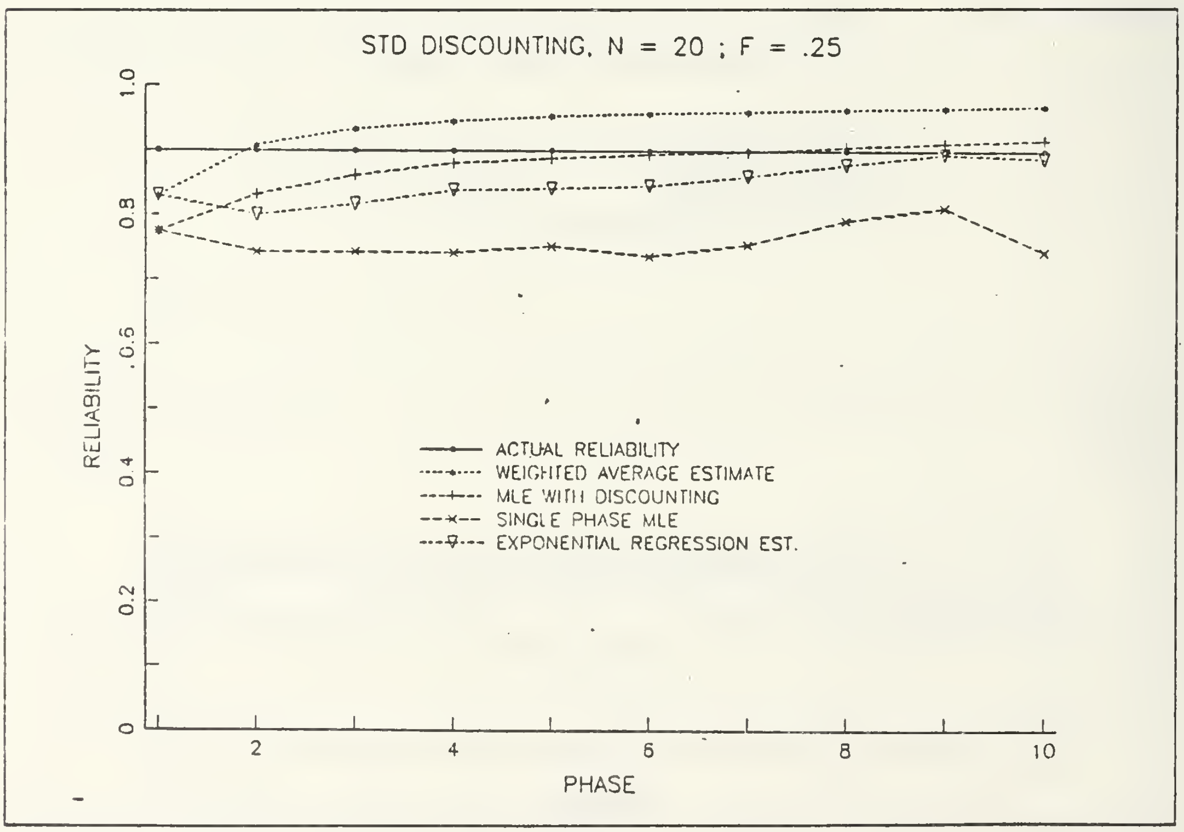


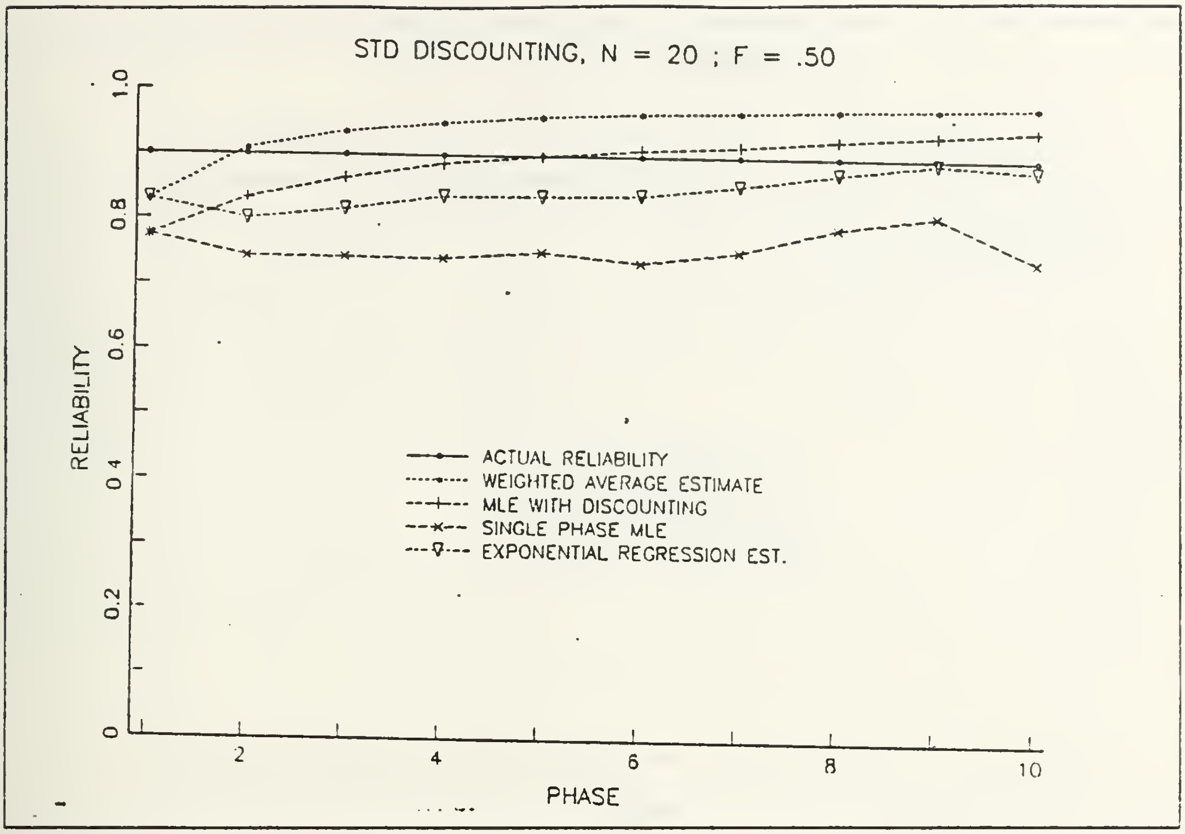
LLOYD DISCOUNTING; GAMMA = .9
ACTUAL RELIABILITY
WEIGHTED AVERAGE ESTIMATE
MLE WITH DISCOUNTING
SINGLE PHASE MLE
EXPONENTIAL REGRESSION EST.
RELIABILITY
0
0.2
0.4
0.6
0.8
1.0
2
4
6
8
10
PHASE

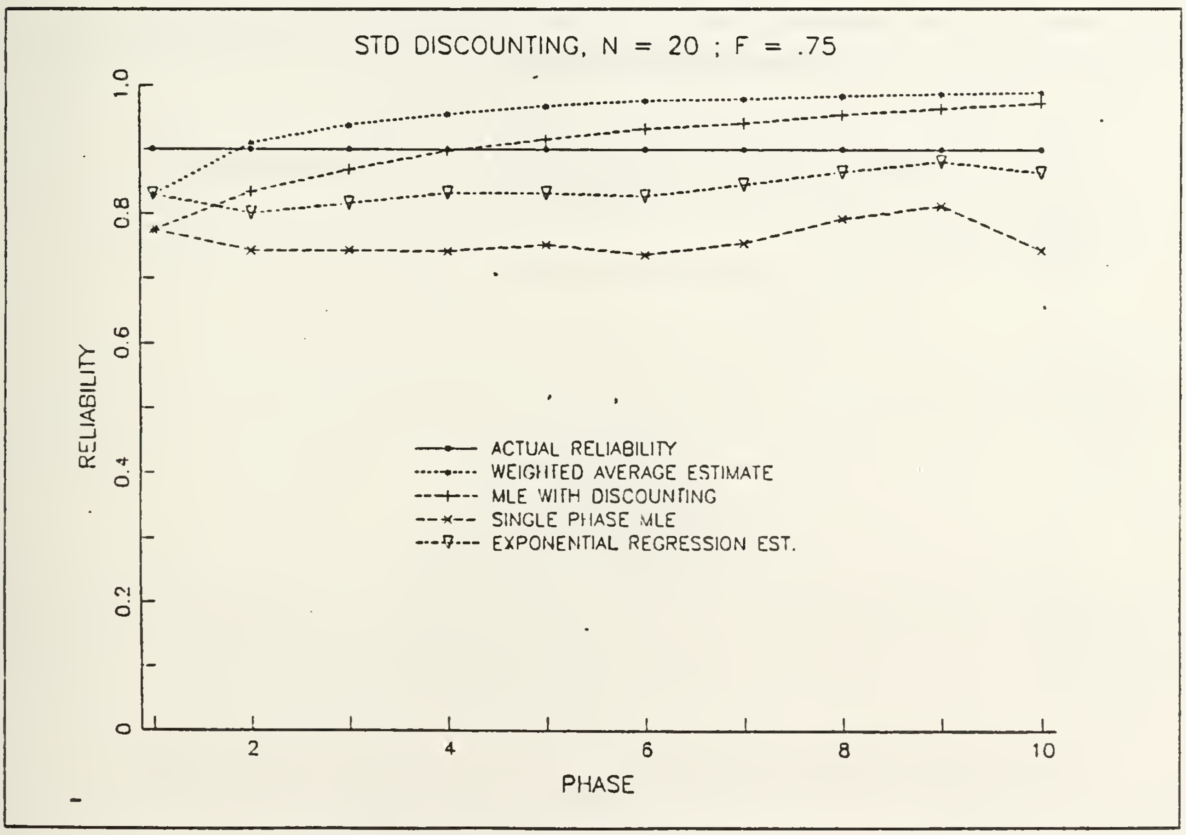

LLOYD DISCOUNTING; GAMMA = .999
ACTUAL RELIABILITY
WEIGHTED AVERAGE ESTIMATE
MLE WITH DISCOUNTING
SINGLE PHASE MLE
EXPONENTIAL REGRESSION EST.
RELIABILITY
0
0.2
0.4
0.6
0.8
1.0
2
4
6
8
10
PHASE

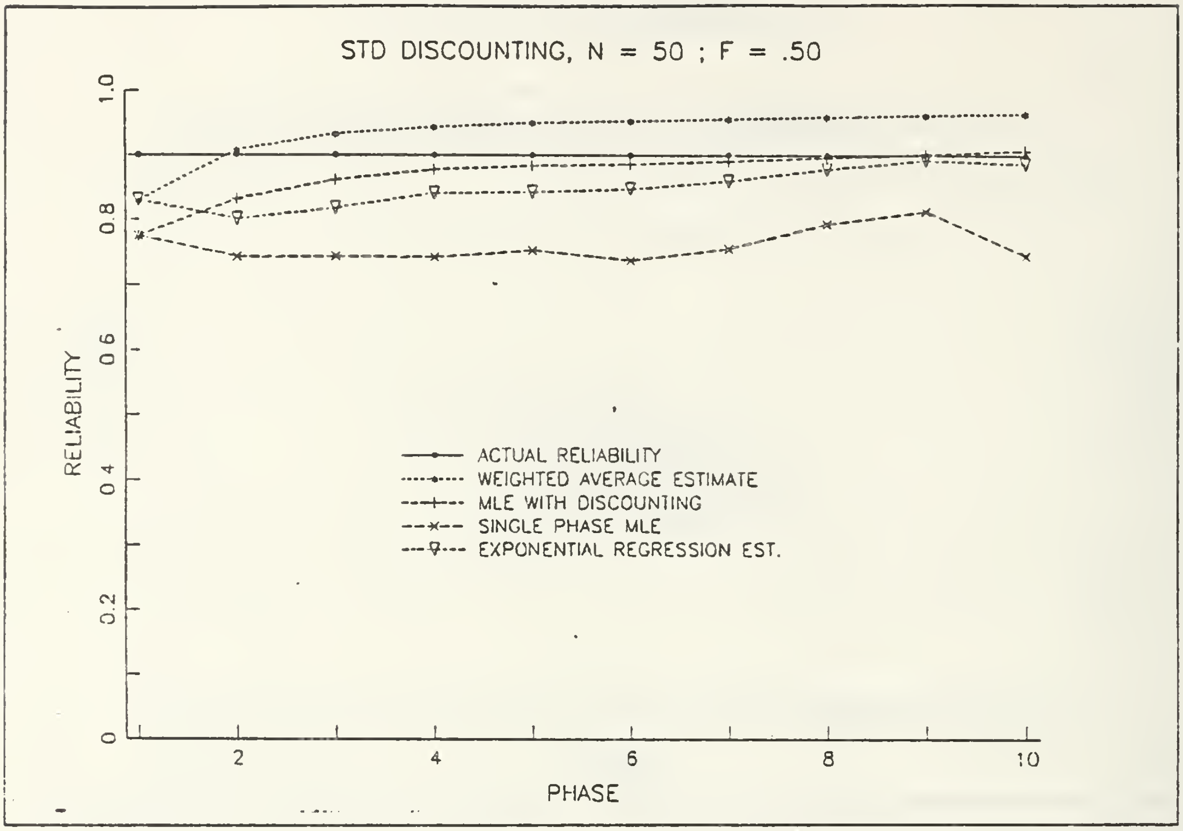

LLOYD DISCOUNTING; GAMMA = .99999
ACTUAL RELIABILITY
WEIGHTED AVERAGE ESTIMATE
MLE WITH DISCOUNTING
SINGLE PHASE MLE
EXPONENTIAL REGRESSION EST.
RELIABILITY
0
0.2
0.4
0.6
0.8
1.0
2
4
6
8
10
PHASE

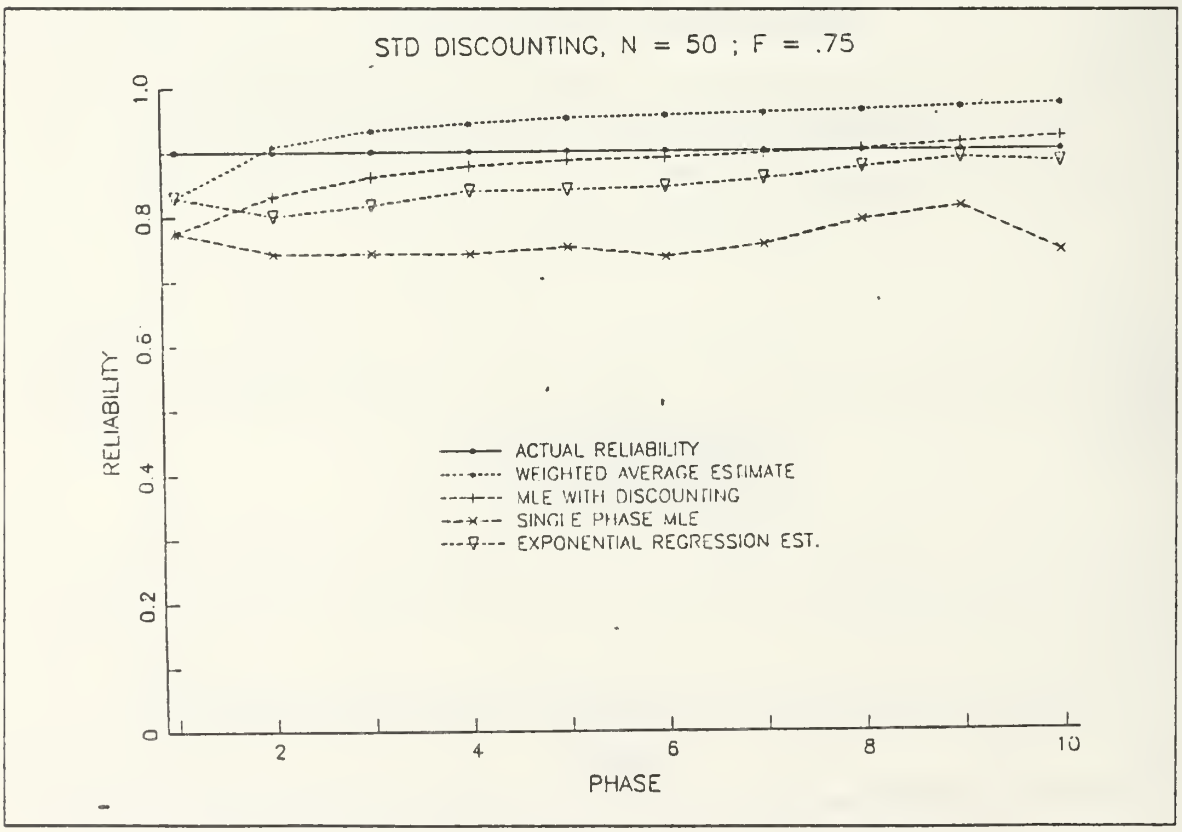

NO DISCOUNTING APPLIED
RELIABILITY
1.0
0.8
0.6
0.4
0.2
0
PHASE
2
4
6
8
10
ACTUAL RELIABILITY
WEIGHTED AVERAGE ESTIMATE
MLE WITH DISCOUNTING
SINGLE PHASE MLE
EXPONENTIAL REGRESSION EST.

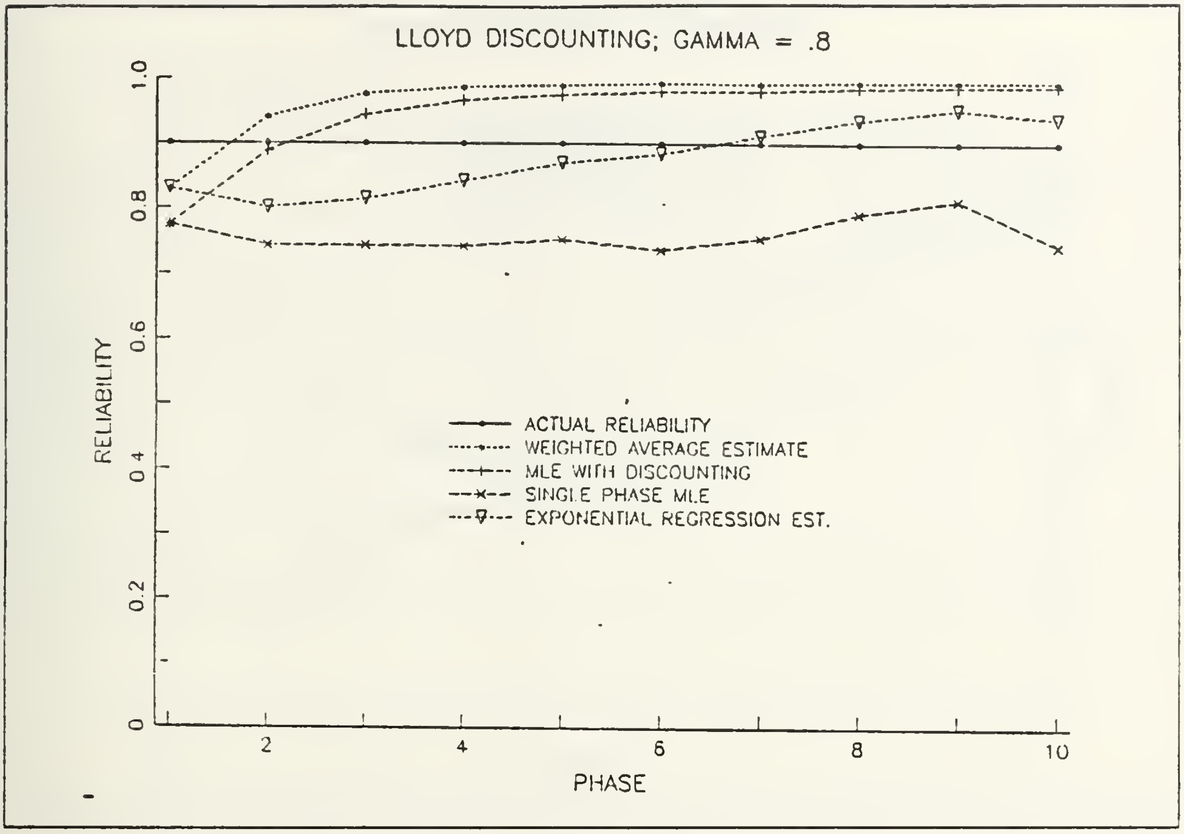

STD DISCOUNTING, N = 20 ; F = .25
RELIABILITY
1.0
0.8
0.6
0.4
0.2
0
PHASE
2
4
6
8
10
ACTUAL RELIABILITY
WEIGHTED AVERAGE ESTIMATE
MLE WITH DISCOUNTING
SINGLE PHASE MLE
EXPONENTIAL REGRESSION EST.

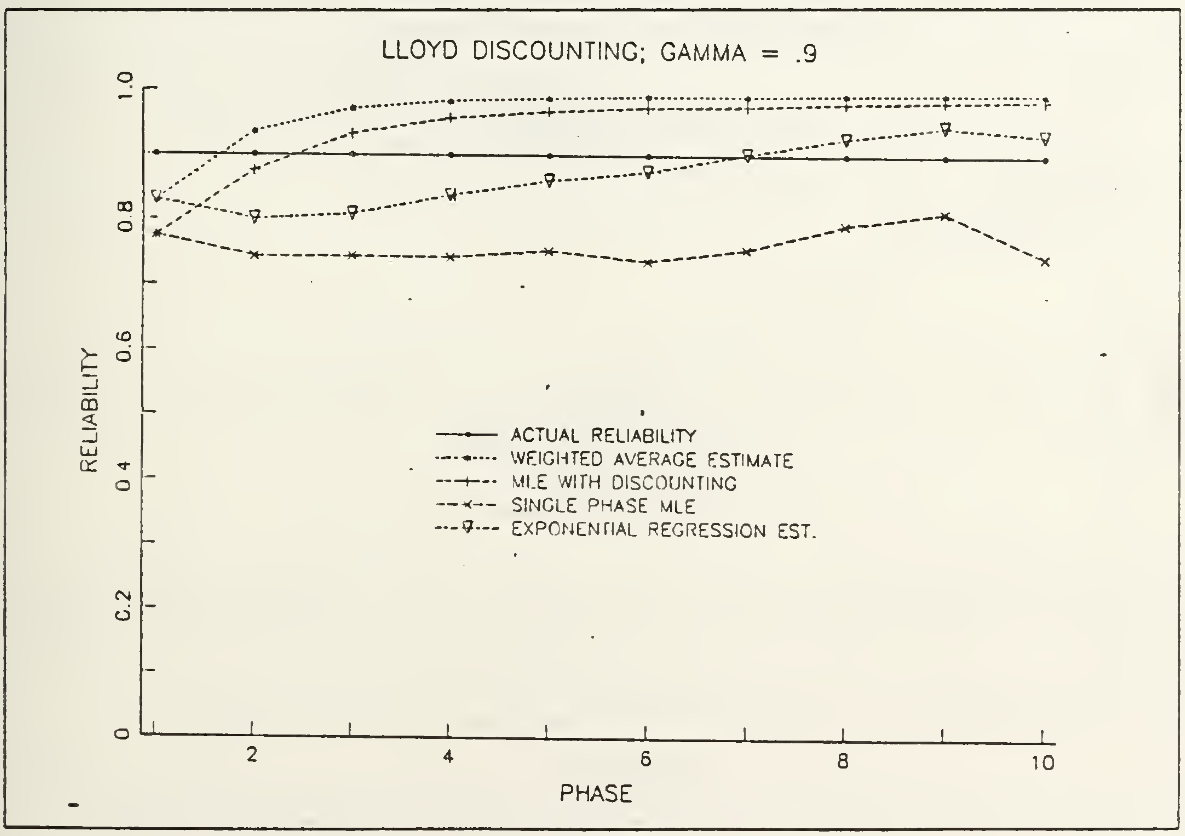

STD DISCOUNTING, N = 20 ; F = .50
RELIABILITY
0
0.2
0.4
0.6
0.8
1.0
2
4
6
8
10
PHASE
ACTUAL RELIABILITY
WEIGHTED AVERAGE ESTIMATE
MLE WITH DISCOUNTING
SINGLE PHASE MLE
EXPONENTIAL REGRESSION EST.

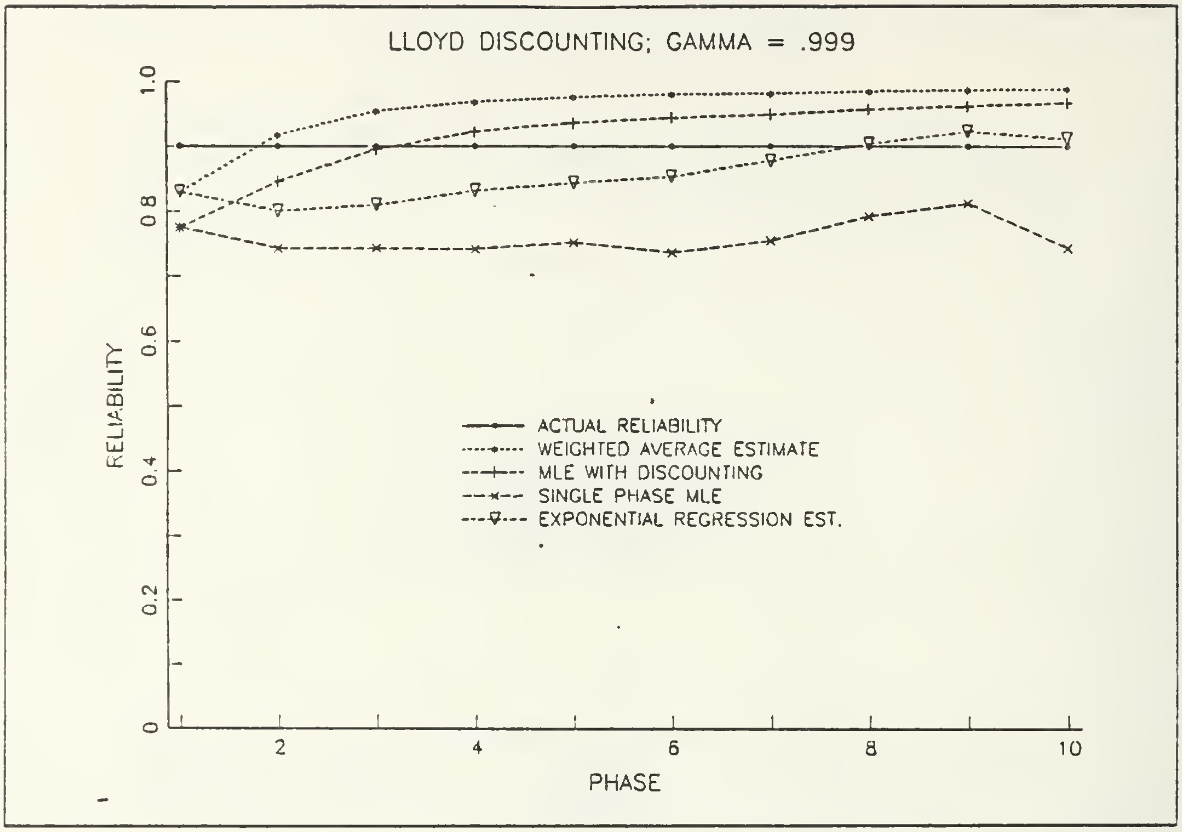

STD DISCOUNTING, N = 20 ; F = .75
RELIABILITY
0
0.2
0.4
0.6
0.8
1.0
2
4
6
8
10
PHASE
ACTUAL RELIABILITY
WEIGHTED AVERAGE ESTIMATE
MLE WITH DISCOUNTING
SINGLE PHASE MLE
EXPONENTIAL REGRESSION EST.

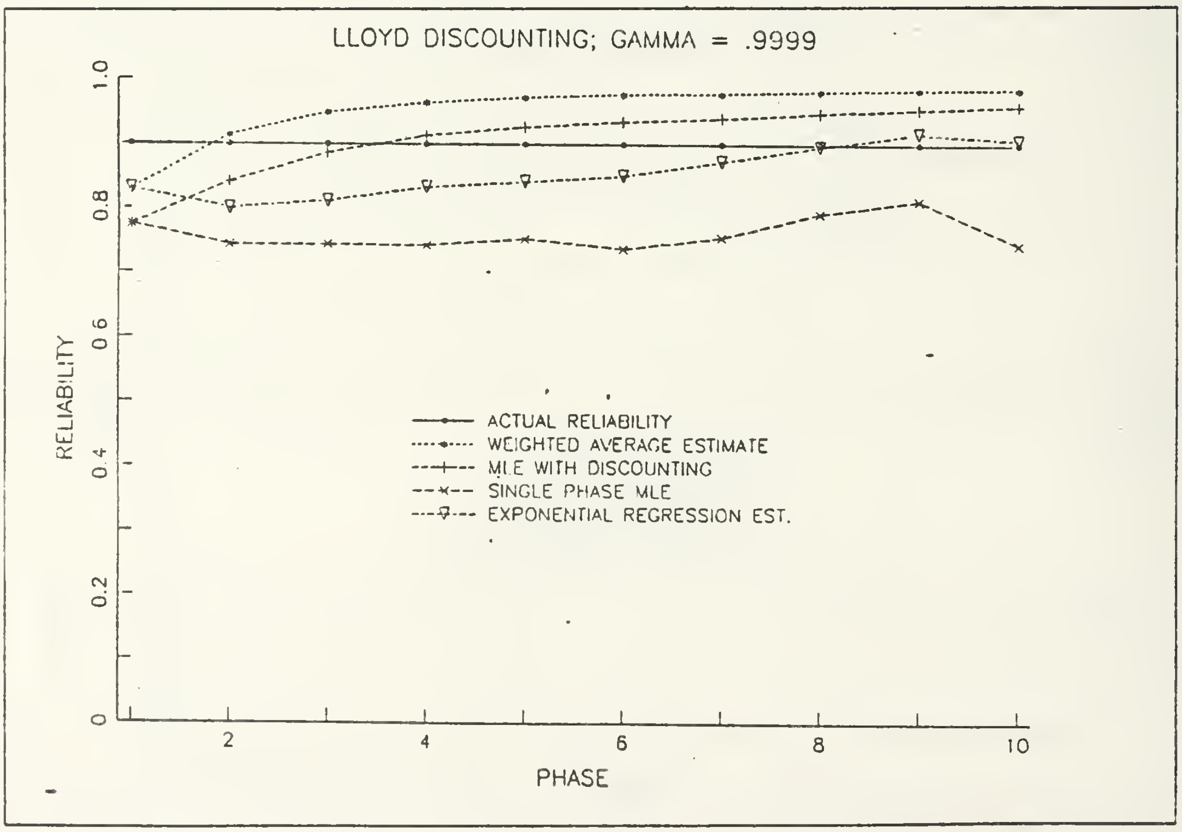

STD DISCOUNTING, N = 50 ; F = .50
RELIABILITY
0
0.2
0.4
0.6
0.8
1.0
2
4
6
8
10
PHASE
ACTUAL RELIABILITY
WEIGHTED AVERAGE ESTIMATE
MLE WITH DISCOUNTING
SINGLE PHASE MLE
EXPONENTIAL REGRESSION EST.

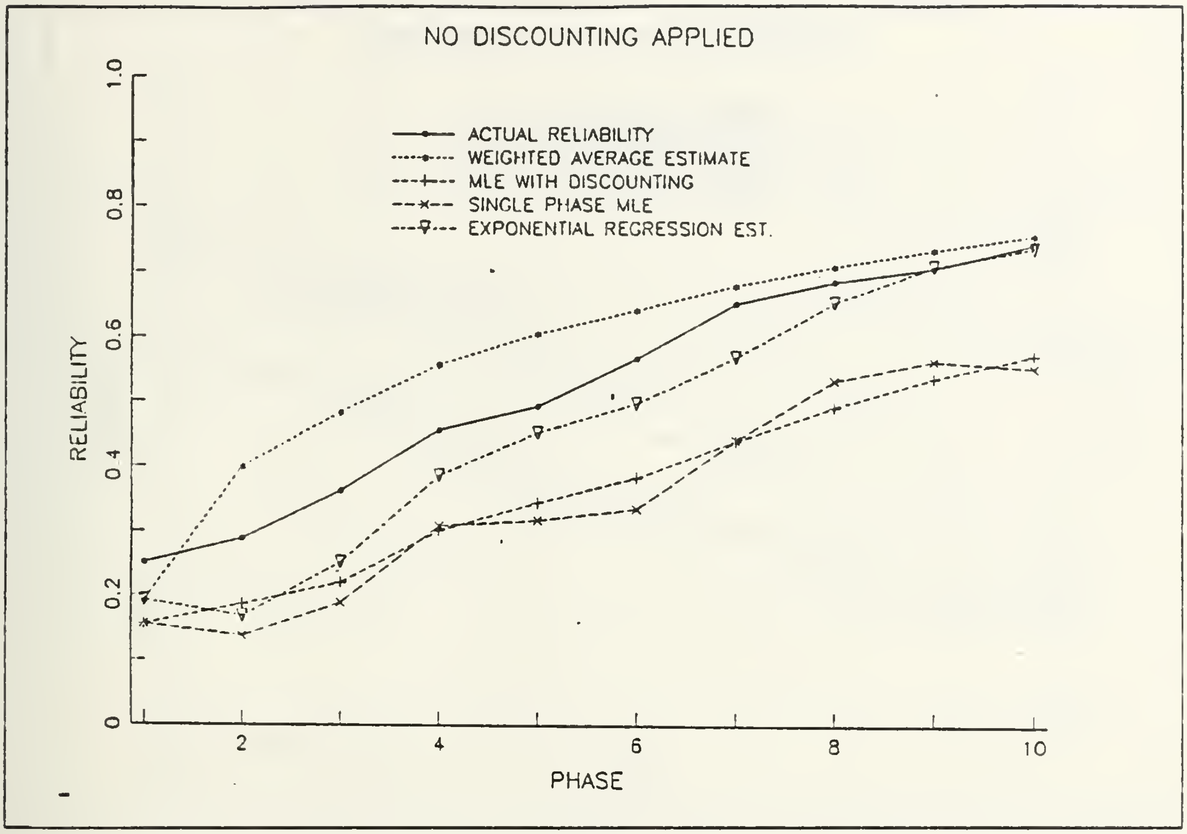

STD DISCOUNTING, N = 50 ; F = .75
RELIABILITY
0
0.2
0.4
0.6
0.8
1.0
2
4
6
8
10
PHASE
ACTUAL RELIABILITY
WEIGHTED AVERAGE ESTIMATE
MLE WITH DISCOUNTING
SINGLE PHASE MLE
EXPONENTIAL REGRESSION EST.

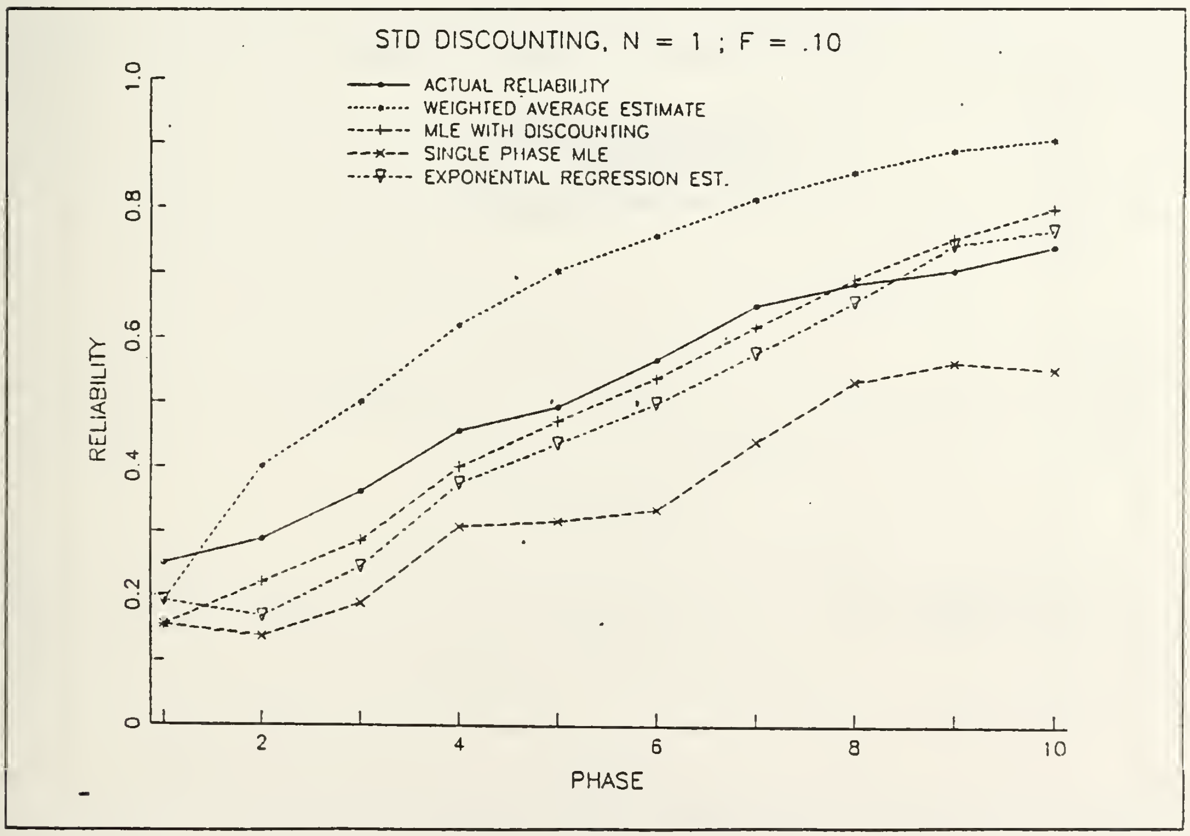

LLOYD DISCOUNTING; GAMMA = .8
RELIABILITY
1.0
0.8
0.6
0.4
0.2
0
2
4
6
8
10
PHASE
ACTUAL RELIABILITY
WEIGHTED AVERAGE ESTIMATE
MLE WITH DISCOUNTING
SINGLE PHASE MLE
EXPONENTIAL REGRESSION EST.

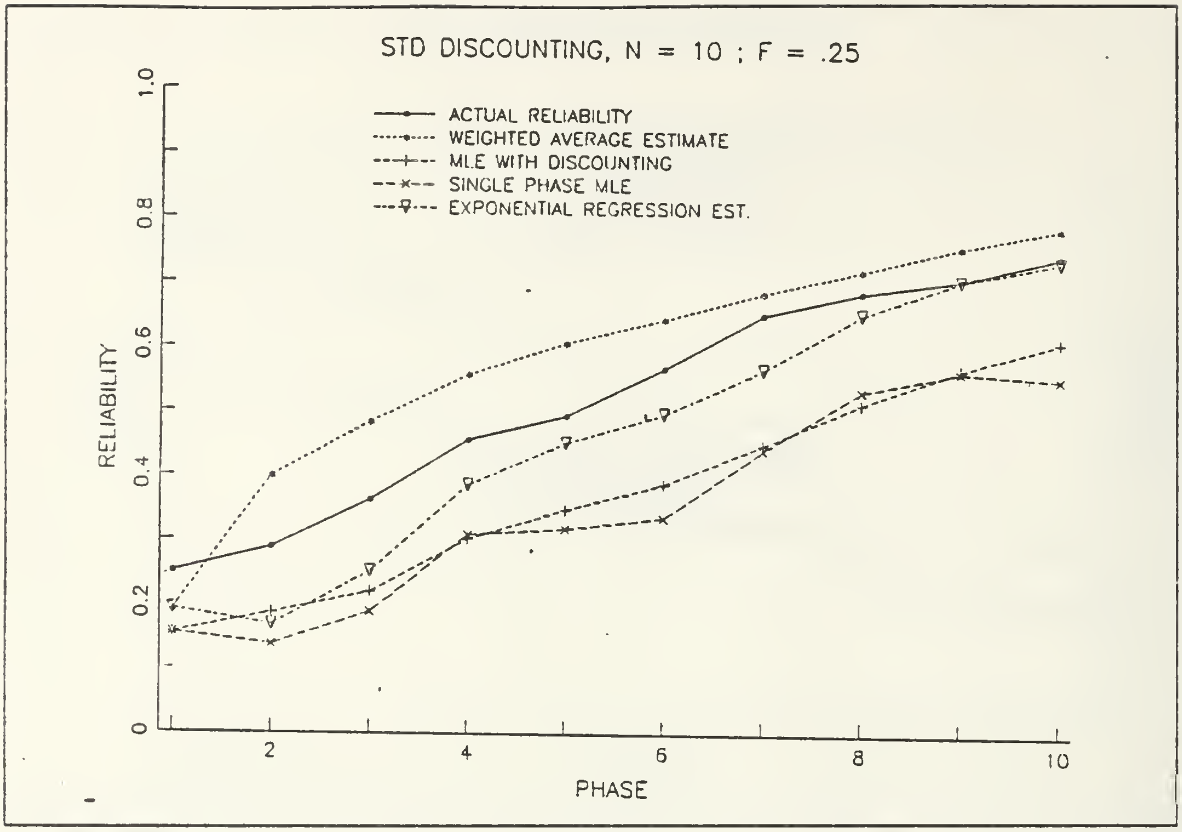

LLOYD DISCOUNTING; GAMMA = .9
RELIABILITY
1.0
0.8
0.6
0.4
0.2
0
2
4
6
8
10
PHASE
ACTUAL RELIABILITY
WEIGHTED AVERAGE ESTIMATE
MLE WITH DISCOUNTING
SINGLE PHASE MLE
EXPONENTIAL REGRESSION EST.


LLOYD DISCOUNTING; GAMMA = .999
RELIABILITY
1.0
0.8
0.6
0.4
0.2
0
2
4
6
8
10
PHASE
ACTUAL RELIABILITY
WEIGHTED AVERAGE ESTIMATE
MLE WITH DISCOUNTING
SINGLE PHASE MLE
EXPONENTIAL REGRESSION EST.

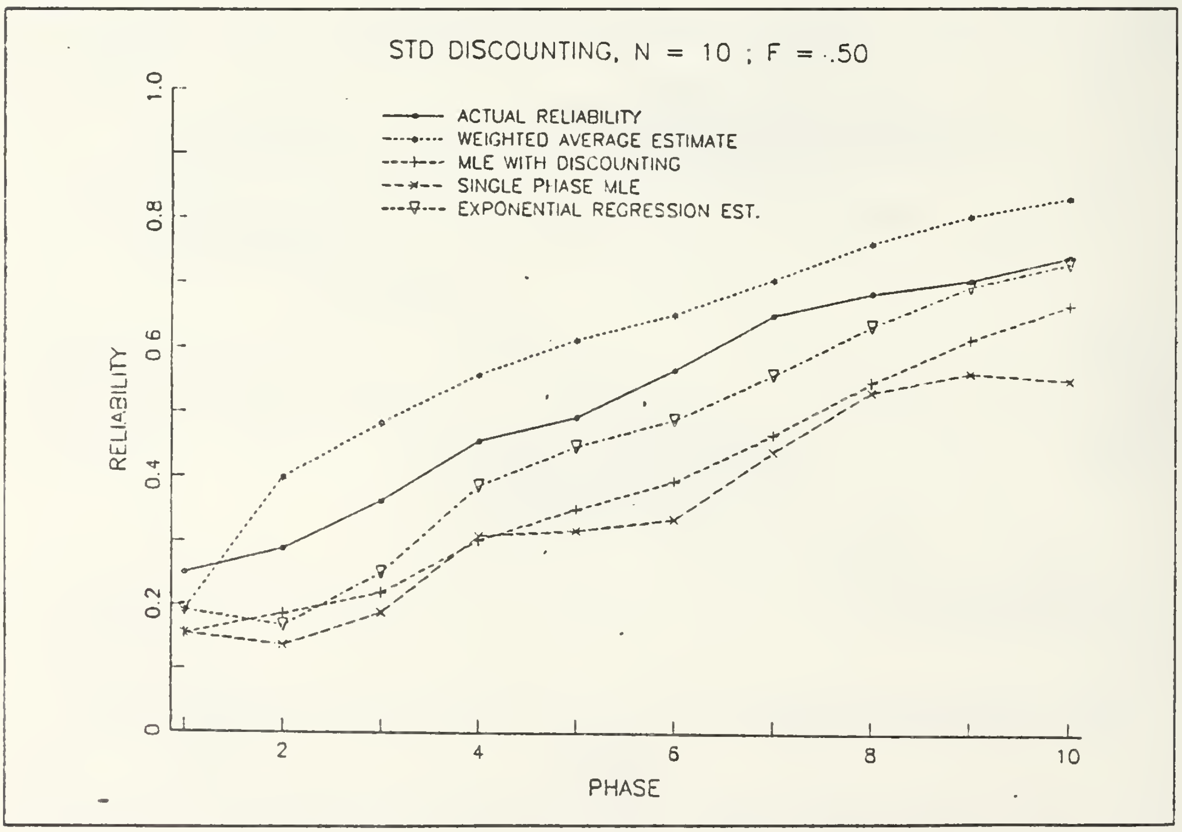

LLOYD DISCOUNTING; GAMMA = .9999
RELIABILITY
1.0
0.8
0.6
0.4
0.2
0
2
4
6
8
10
PHASE
ACTUAL RELIABILITY
WEIGHTED AVERAGE ESTIMATE
MLE WITH DISCOUNTING
SINGLE PHASE MLE
EXPONENTIAL REGRESSION EST.


NO DISCOUNTING APPLIED
ACTUAL RELIABILITY
WEIGHTED AVERAGE ESTIMATE
MLE WITH DISCOUNTING
SINGLE PHASE MLE
EXPONENTIAL REGRESSION EST.
RELIABILITY
0
0.2
0.4
0.6
0.8
1.0
2
4
6
8
10
PHASE

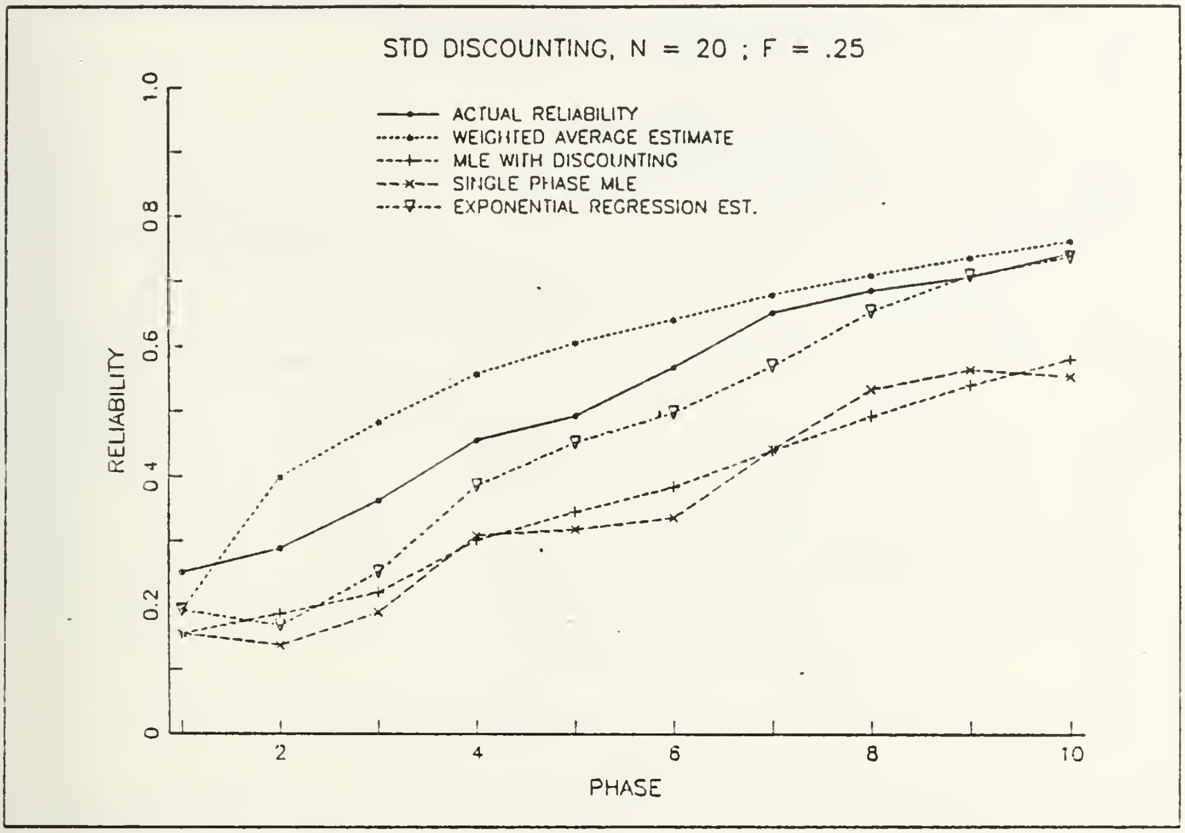

STD DISCOUNTING, N = 1 ; F = .10
ACTUAL RELIABILITY
WEIGHTED AVERAGE ESTIMATE
MLE WITH DISCOUNTING
SINGLE PHASE MLE
EXPONENTIAL REGRESSION EST.
RELIABILITY
0
0.2
0.4
0.6
0.8
1.0
2
4
6
8
10
PHASE



STD DISCOUNTING, N = 10 ; F = .25
ACTUAL RELIABILITY
WEIGHTED AVERAGE ESTIMATE
MLE WITH DISCOUNTING
SINGLE PHASE MLE
EXPONENTIAL REGRESSION EST.
RELIABILITY
0
0.2
0.4
0.6
0.8
1.0
2
4
6
8
10
PHASE



STD DISCOUNTING, N = 10 ; F = .50
ACTUAL RELIABILITY
WEIGHTED AVERAGE ESTIMATE
MLE WITH DISCOUNTING
SINGLE PHASE MLE
EXPONENTIAL REGRESSION EST.
RELIABILITY
0
0.2
0.4
0.6
0.8
1.0
2
4
6
8
10
PHASE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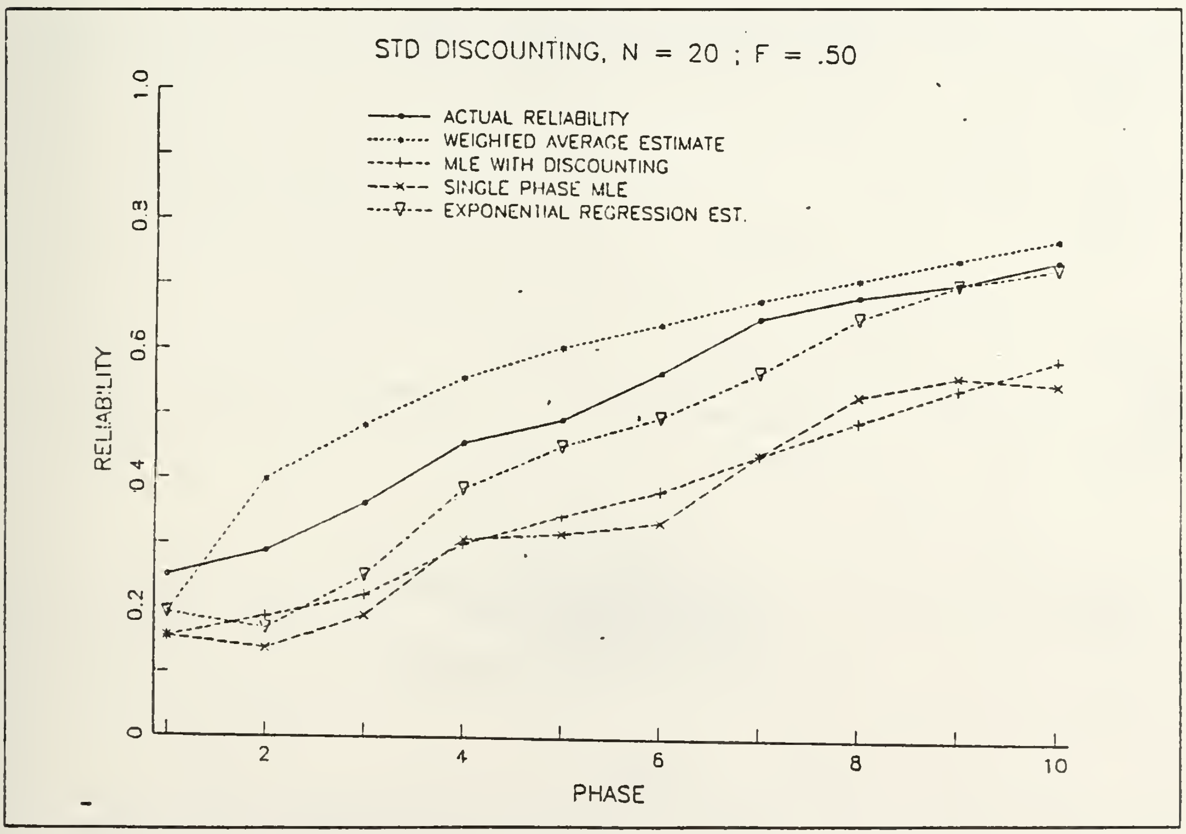
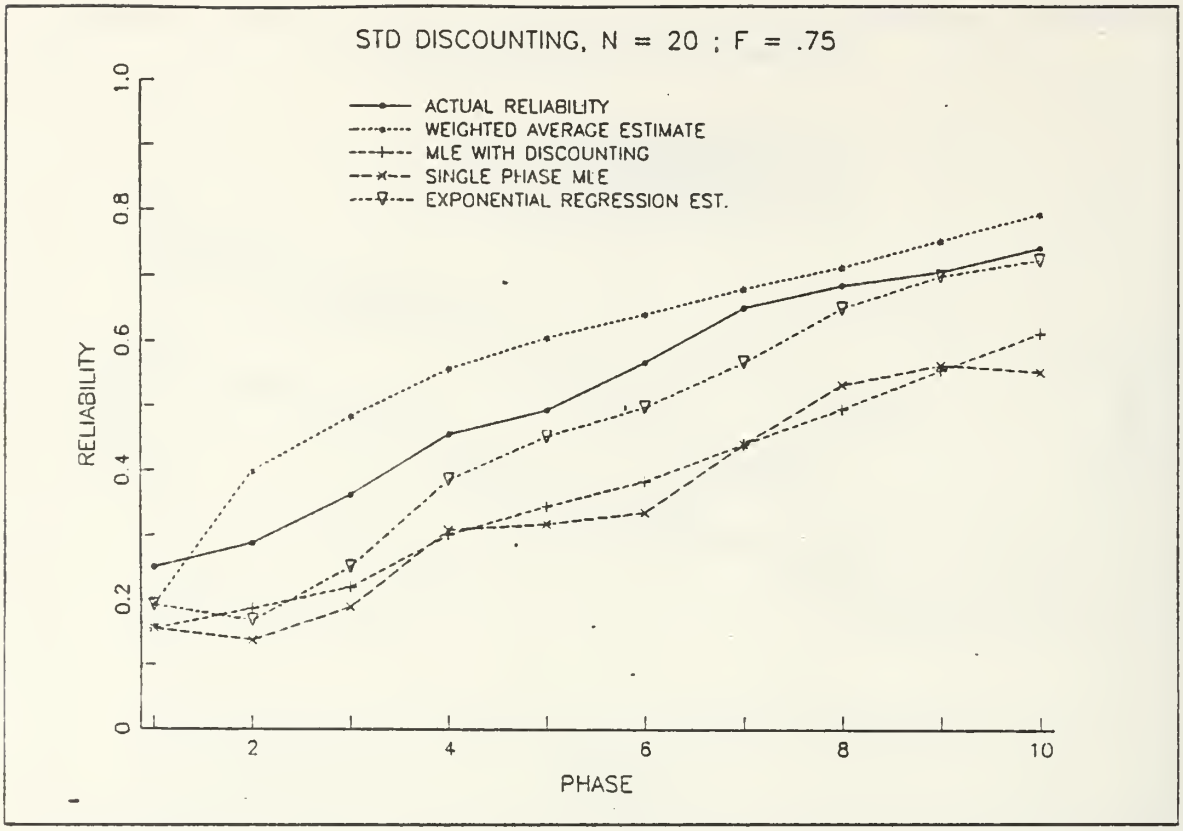
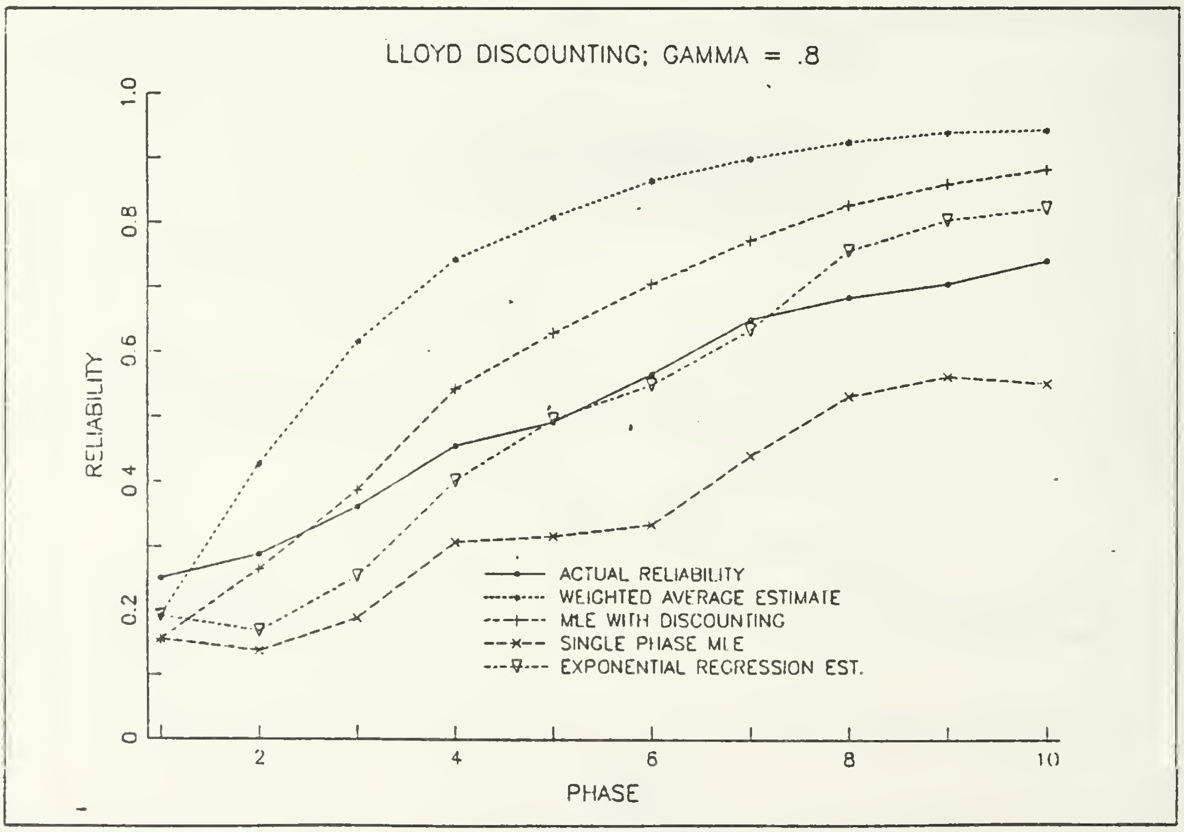
ACTUAL RELIABILITY
WEIGHTED AVERAGE ESTIMATE
MLE WITH DISCOUNTING
SINGLE PHASE MLE
EXPONENTIAL REGRESSION EST.
RELIABILITY
PHASE
0
0.2
0.4
0.6
0.8
1.0
2
4
6
8
10

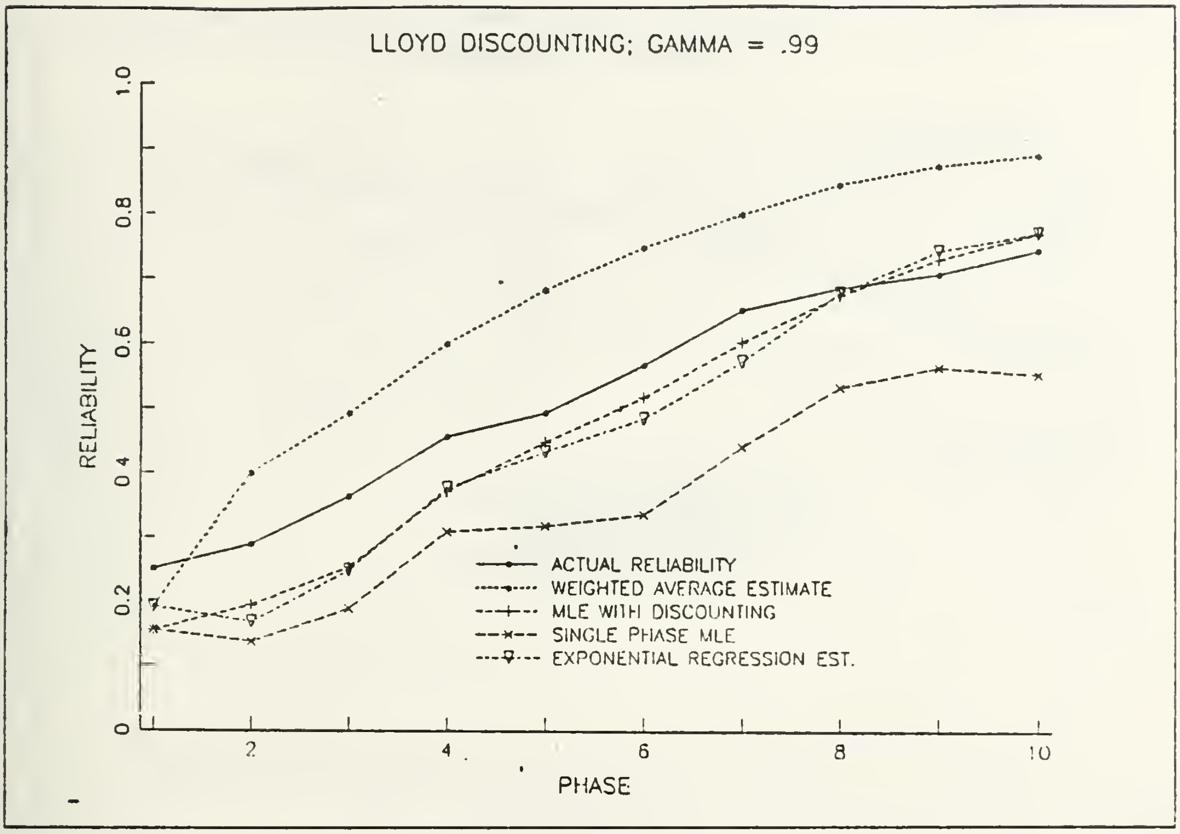

STD DISCOUNTING, N = 20 ; F = .50
ACTUAL RELIABILITY
WEIGHTED AVERAGE ESTIMATE
MLE WITH DISCOUNTING
SINGLE PHASE MLE
EXPONENTIAL REGRESSION EST.
RELIABILITY
PHASE
0
0.2
0.4
0.6
0.8
1.0
2
4
6
8
10

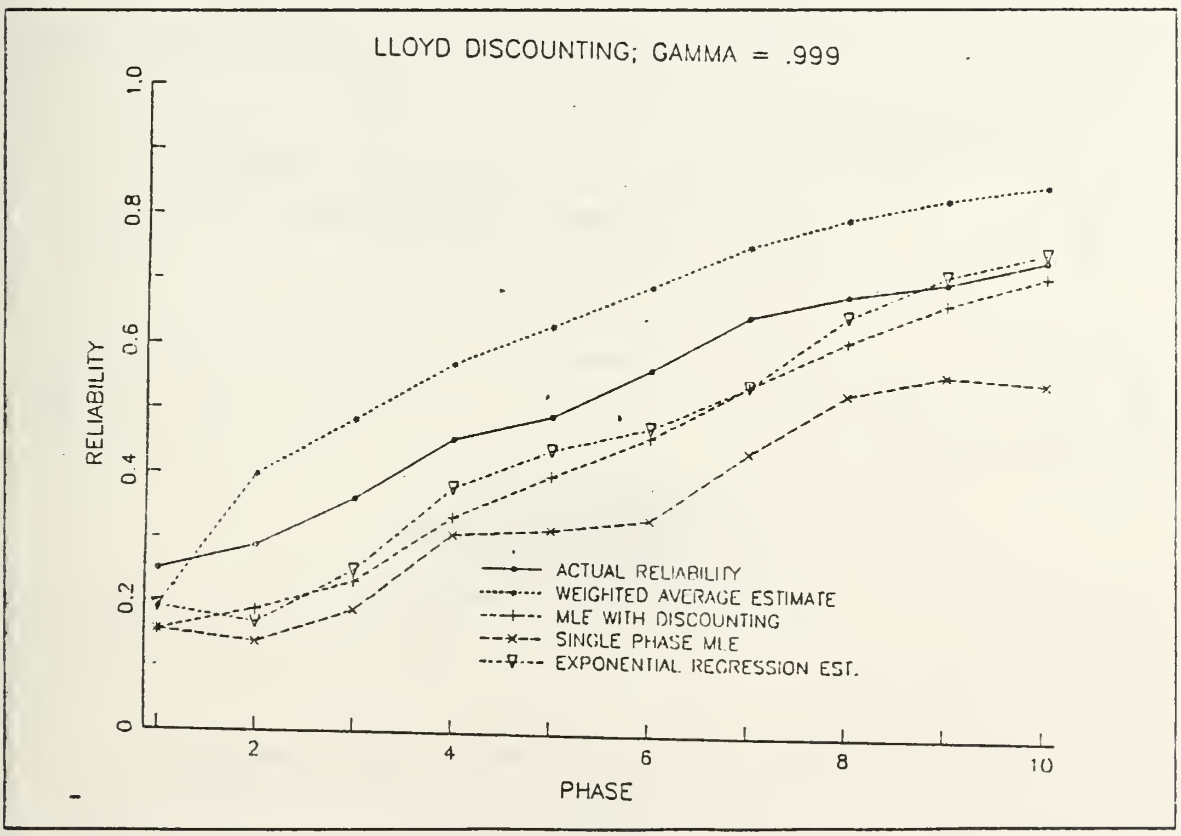


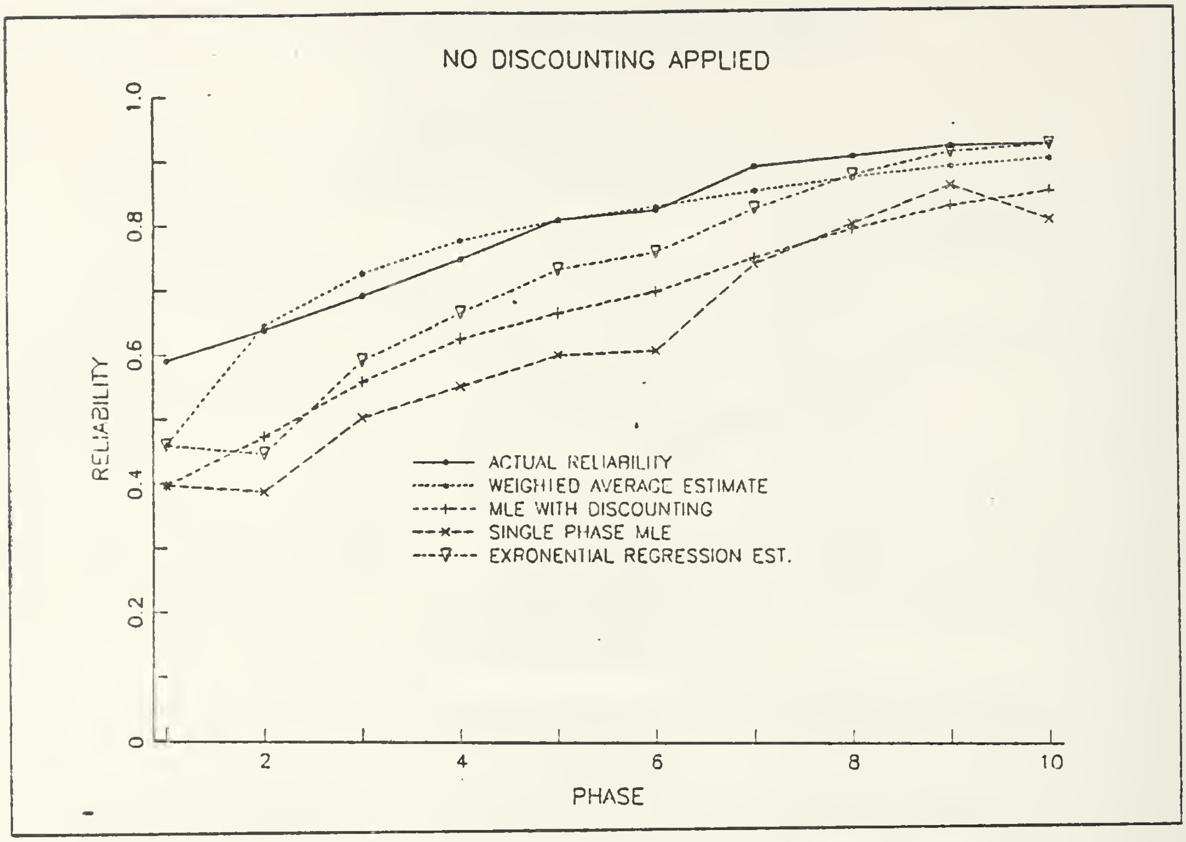
STD DISCOUNTING, N = 20 ; F = .75
ACTUAL RELIABILITY
WEIGHTED AVERAGE ESTIMATE
MLE WITH DISCOUNTING
SINGLE PHASE MLE
EXPONENTIAL REGRESSION EST.
RELIABILITY
0
0.2
0.4
0.6
0.8
1.0
2
4
6
8
10
PHASE

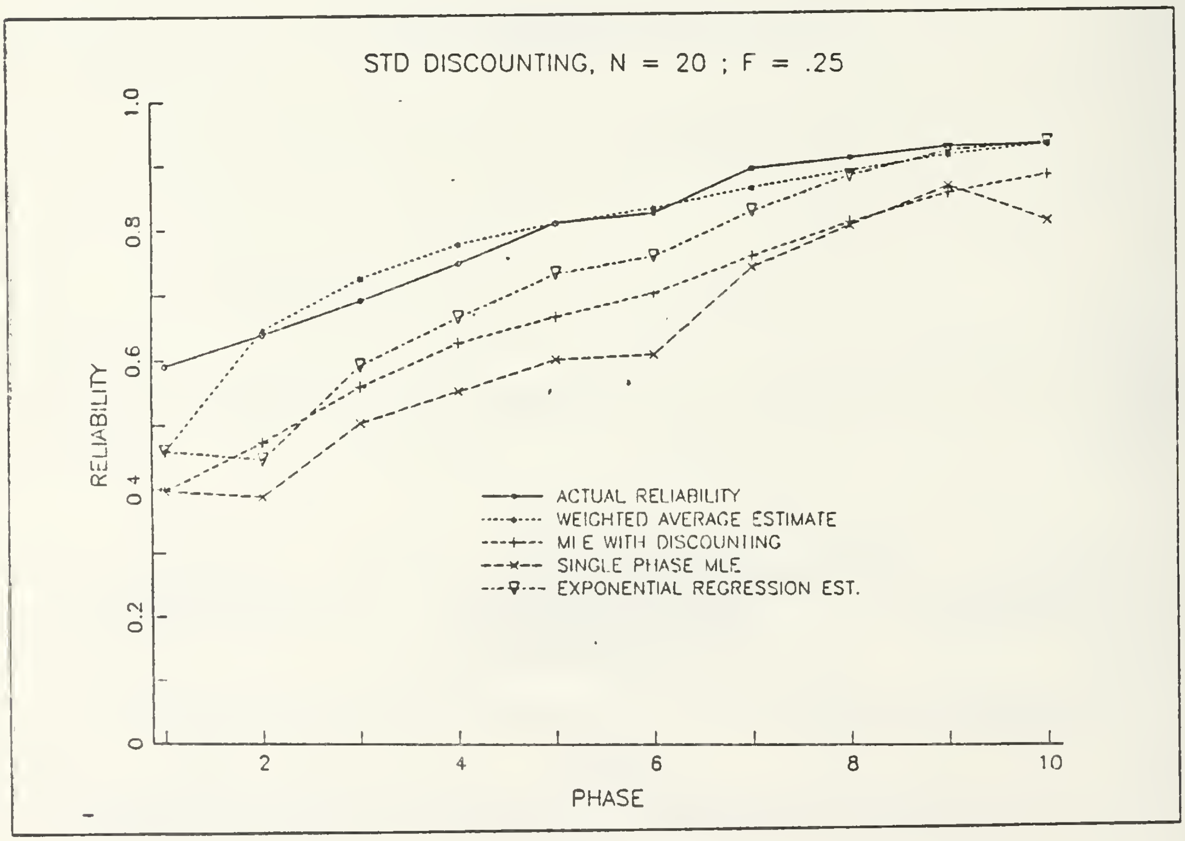


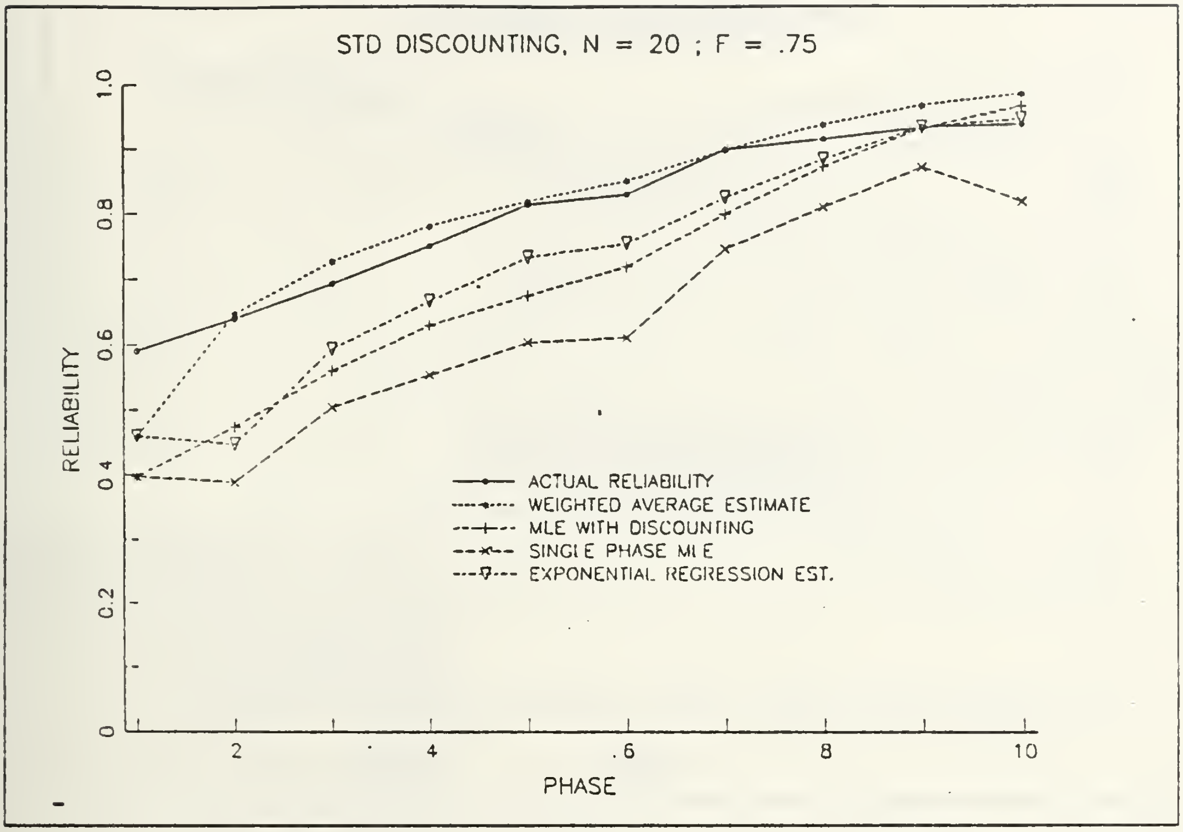
LLOYD DISCOUNTING; GAMMA = .8
ACTUAL RELIABILITY
WEIGHTED AVERAGE ESTIMATE
MLE WITH DISCOUNTING
SINGLE PHASE MLE
EXPONENTIAL REGRESSION EST.
RELIABILITY
0
0.2
0.4
0.6
0.8
1.0
2
4
6
8
10
PHASE

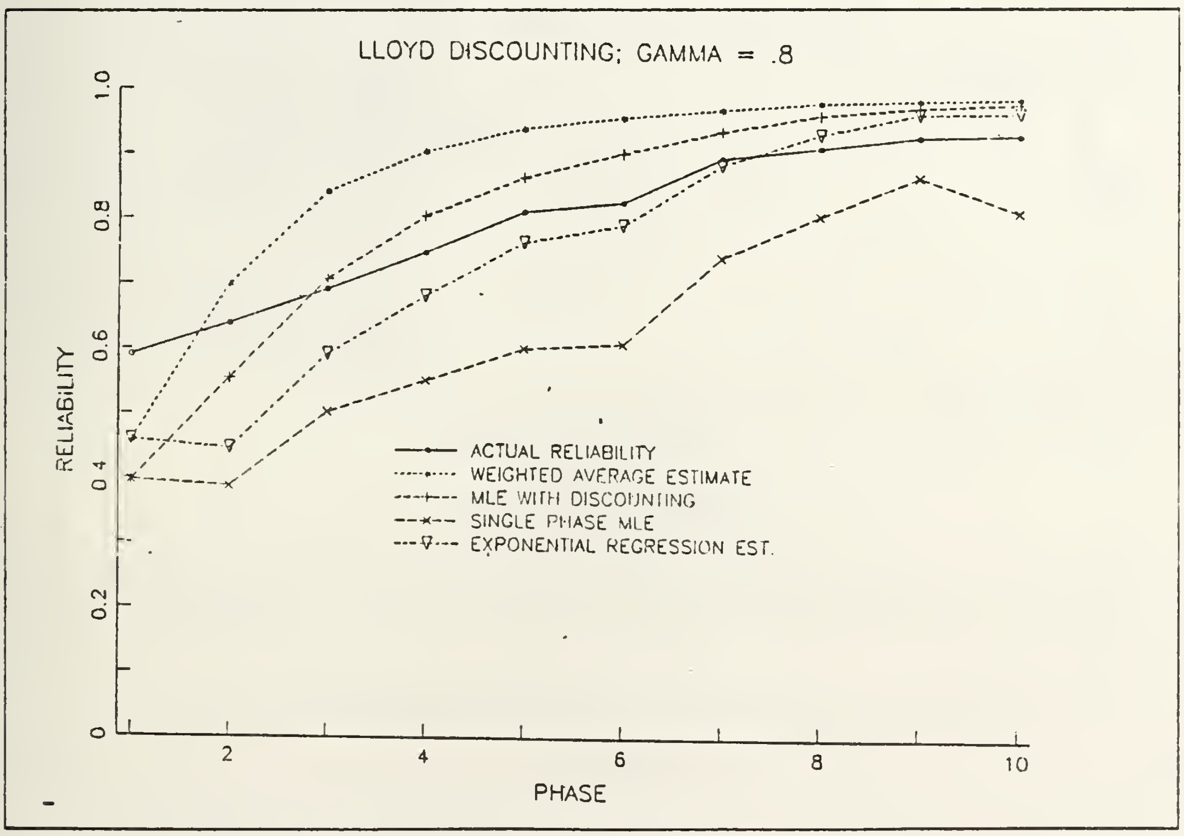

LLOYD DISCOUNTING; GAMMA = .99
RELIABILITY
0
0.2
0.4
0.6
0.8
1.0
2
4
6
8
10
PHASE
ACTUAL RELIABILITY
WEIGHTED AVERAGE ESTIMATE
MLE WITH DISCOUNTING
SINGLE PHASE MLE
EXPONENTIAL REGRESSION EST.

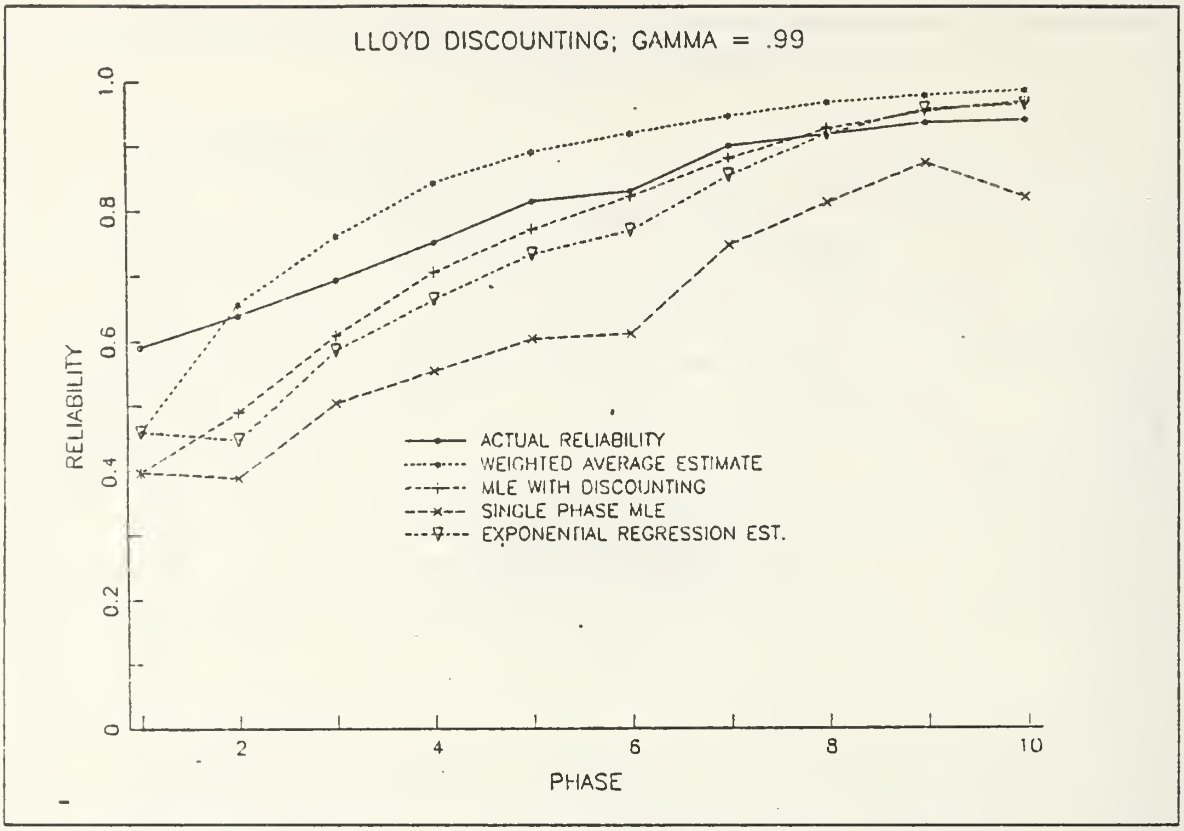

LLOYD DISCOUNTING; GAMMA = .999
RELIABILITY
0
0.2
0.4
0.6
0.8
1.0
2
4
6
8
10
PHASE
ACTUAL RELIABILITY
WEIGHTED AVERAGE ESTIMATE
MLE WITH DISCOUNTING
SINGLE PHASE MLE
EXPONENTIAL REGRESSION EST.


NO DISCOUNTING APPLIED
RELIABILITY
0
0.2
0.4
0.6
0.8
1.0
2
4
6
8
10
PHASE
ACTUAL RELIABILITY
WEIGHTED AVERAGE ESTIMATE
MLE WITH DISCOUNTING
SINGLE PHASE MLE
EXPONENTIAL REGRESSION EST.

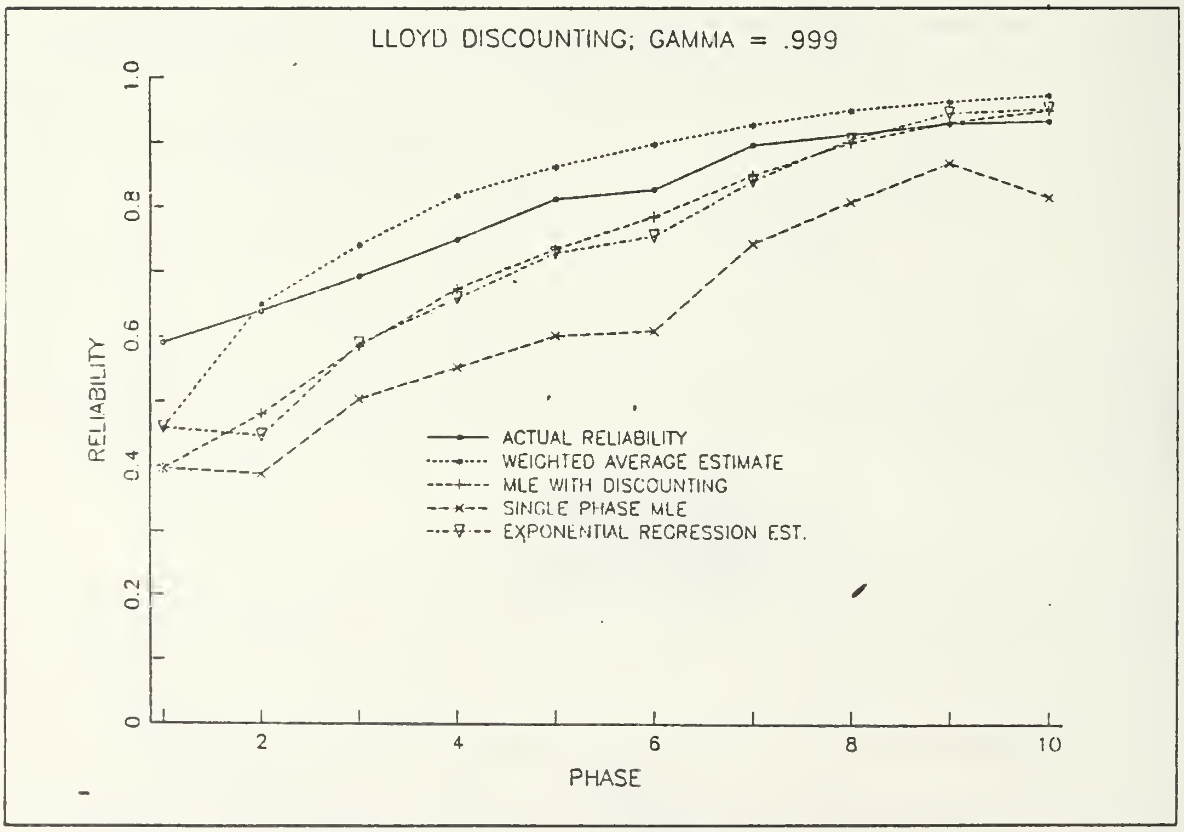

STD DISCOUNTING, N = 20 ; F = .25
RELIABILITY
0
0.2
0.4
0.6
0.8
1.0
2
4
6
8
10
PHASE
ACTUAL RELIABILITY
WEIGHTED AVERAGE ESTIMATE
MLE WITH DISCOUNTING
SINGLE PHASE MLE
EXPONENTIAL REGRESSION EST.


STD DISCOUNTING, N = 20 ; F = .75
RELIABILITY
0
0.2
0.4
0.6
0.8
1.0
2
4
6
8
10
PHASE
ACTUAL RELIABILITY
WEIGHTED AVERAGE ESTIMATE
MLE WITH DISCOUNTING
SINGLE PHASE MLE
EXPONENTIAL REGRESSION EST.

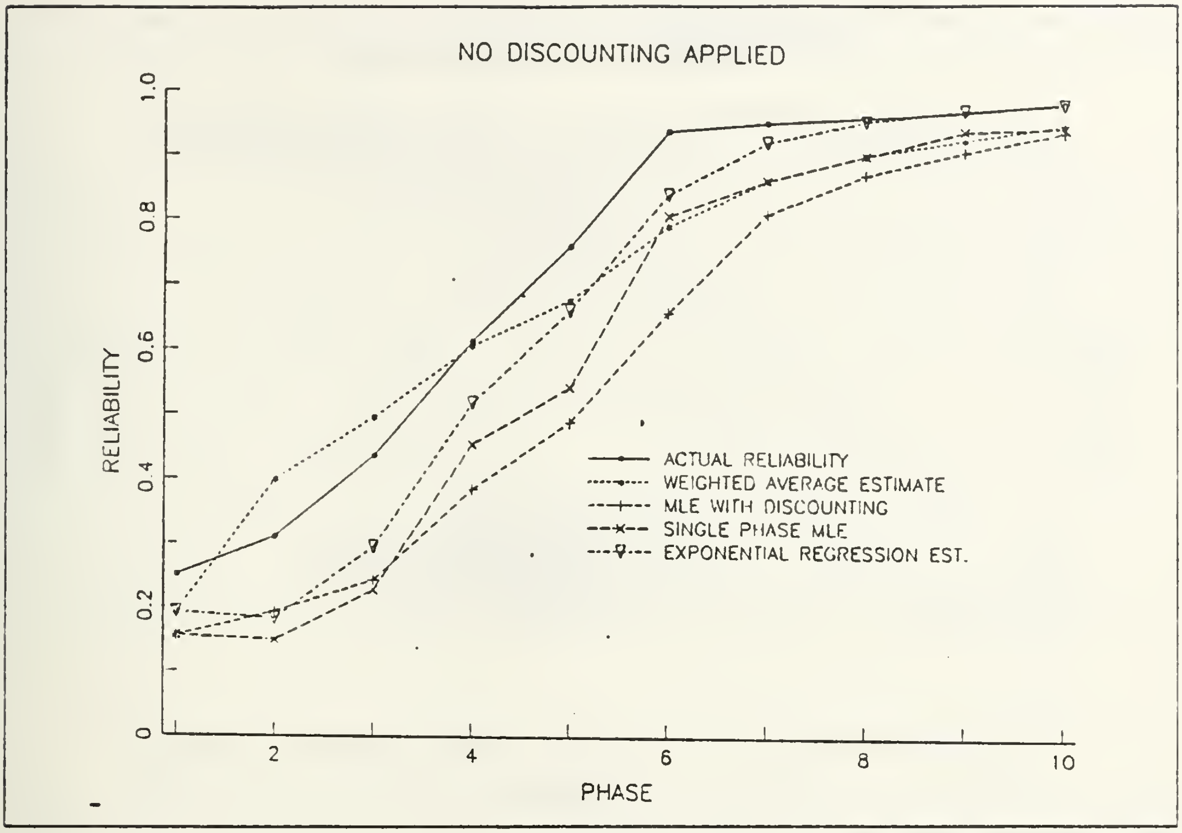

LLOYD DISCOUNTING; GAMMA = .8
RELIABILITY
0
0.2
0.4
0.6
0.8
1.0
2
4
6
8
10
PHASE
ACTUAL RELIABILITY
WEIGHTED AVERAGE ESTIMATE
MLE WITH DISCOUNTING
SINGLE PHASE MLE
EXPONENTIAL REGRESSION EST.

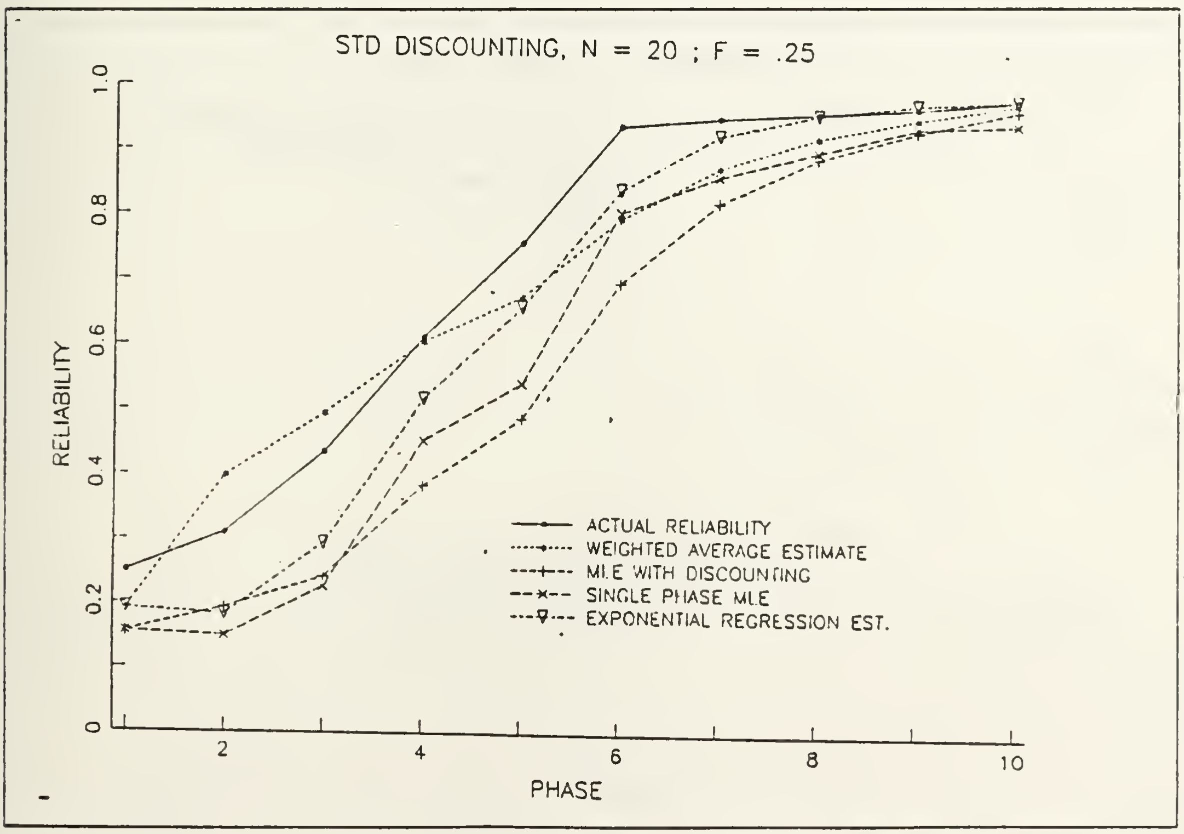


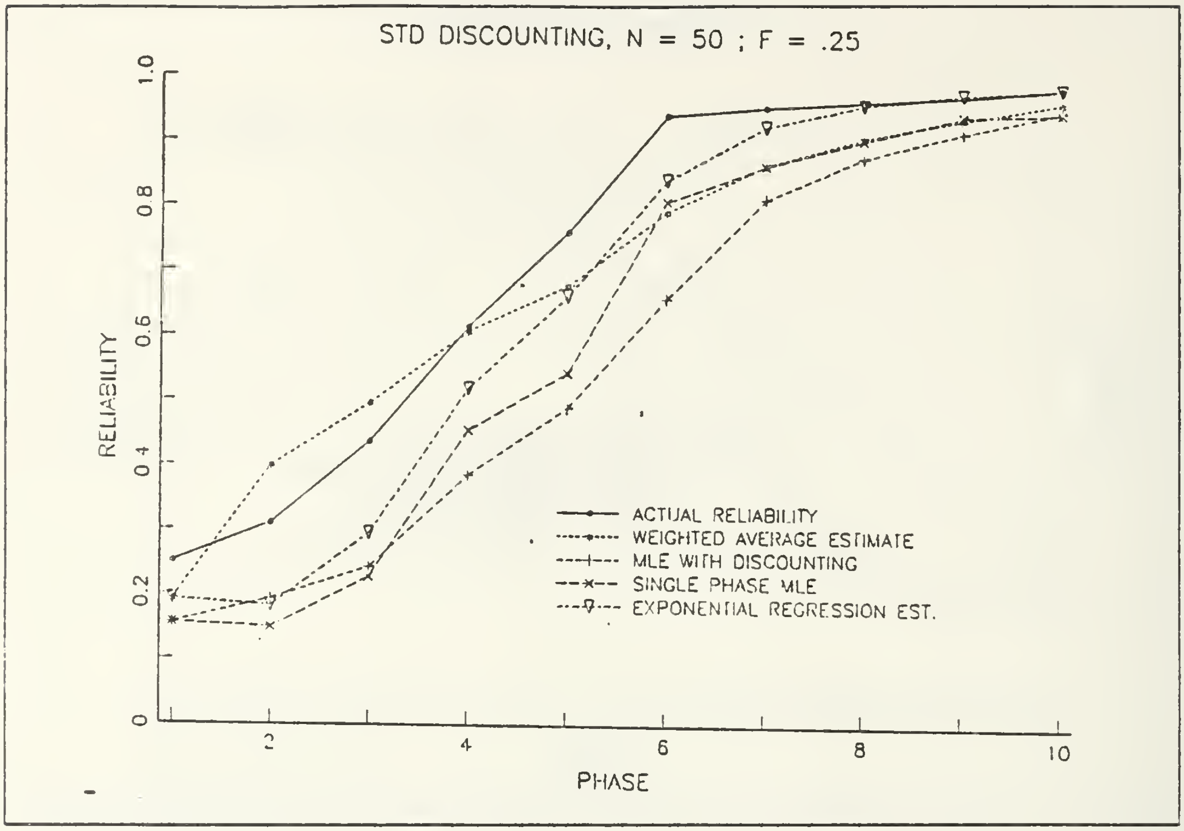
LLOYD DISCOUNTING; GAMMA = .99
RELIABILITY
0
0.2
0.4
0.6
0.8
1.0
2
4
6
8
10
PHASE
ACTUAL RELIABILITY
WEIGHTED AVERAGE ESTIMATE
MLE WITH DISCOUNTING
SINGLE PHASE MLE
EXPONENTIAL REGRESSION EST.

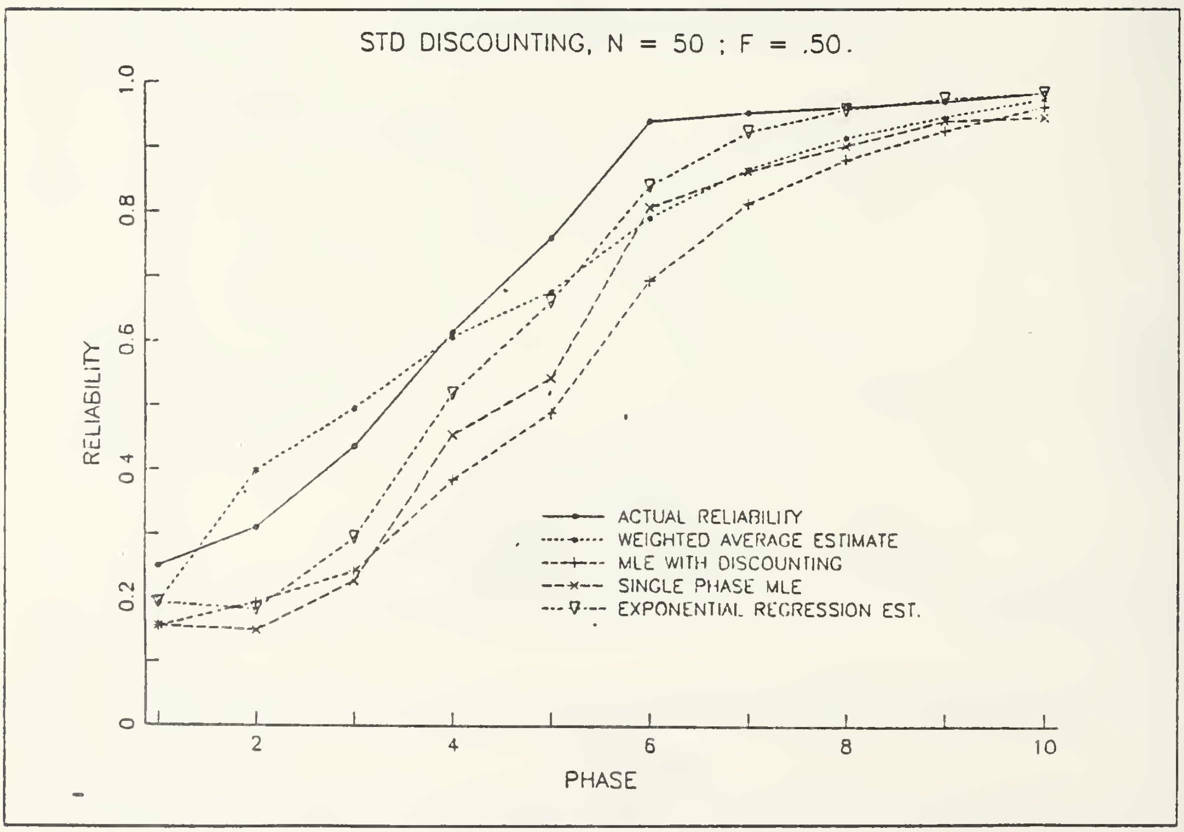


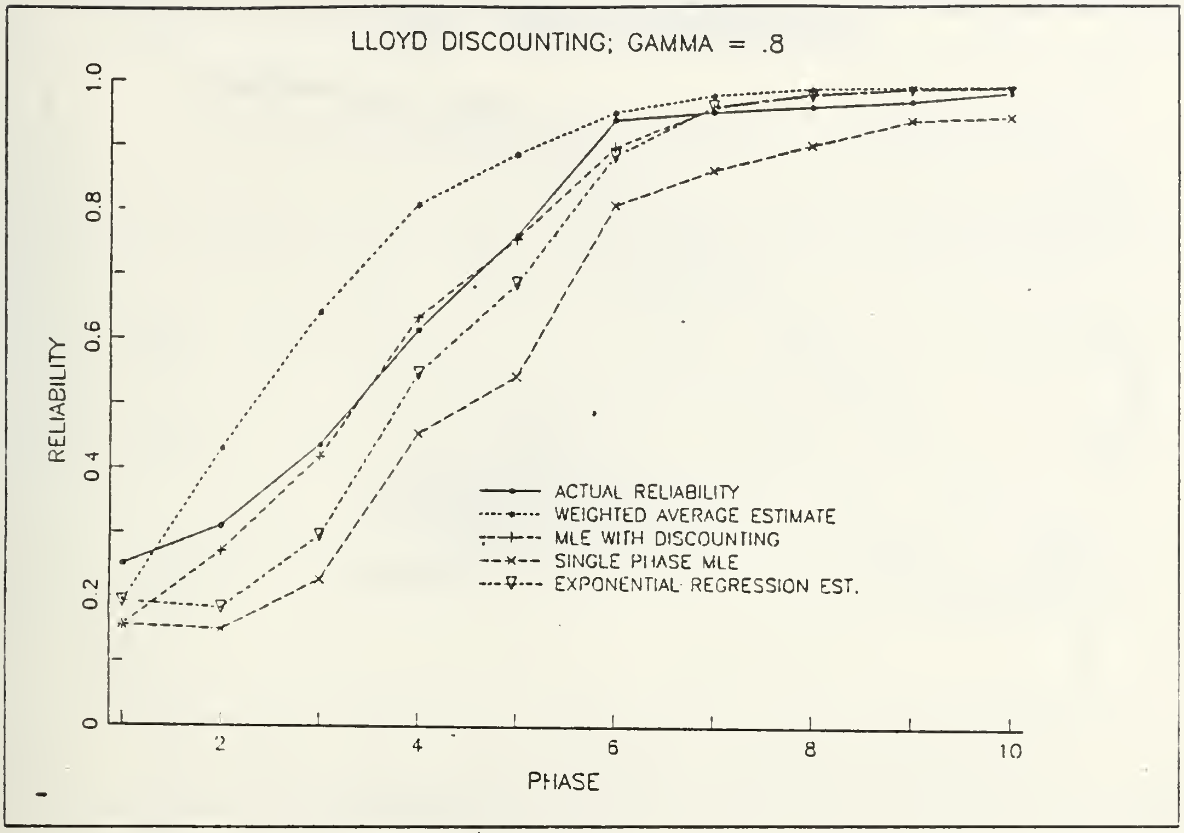
LLOYD DISCOUNTING; GAMMA = .999
RELIABILITY
0
0.2
0.4
0.6
0.8
1.0
2
4
6
8
10
PHASE
ACTUAL RELIABILITY
WEIGHTED AVERAGE ESTIMATE
MLE WITH DISCOUNTING
SINGLE PHASE MLE
EXPONENTIAL REGRESSION EST.

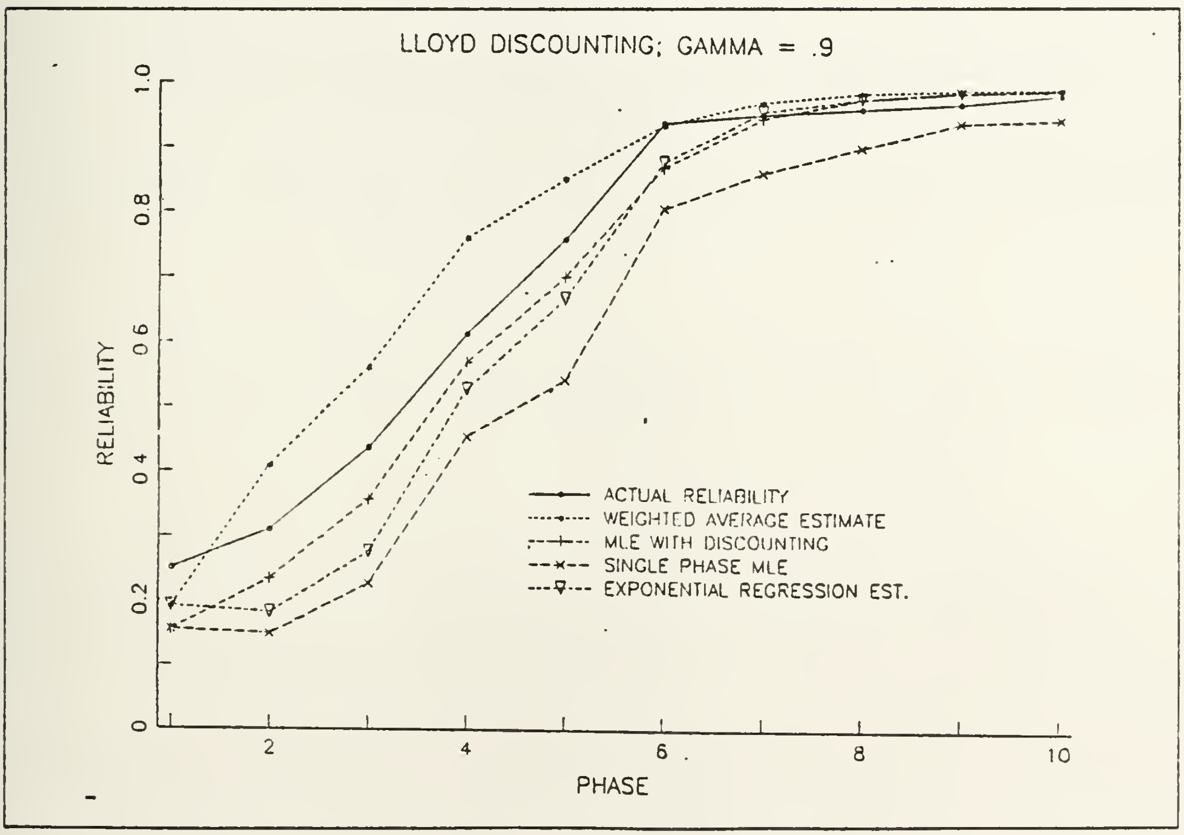

NO DISCOUNTING APPLIED
RELIABILITY
PHASE
ACTUAL RELIABILITY
WEIGHTED AVERAGE ESTIMATE
MLE WITH DISCOUNTING
SINGLE PHASE MLE
EXPONENTIAL REGRESSION EST.

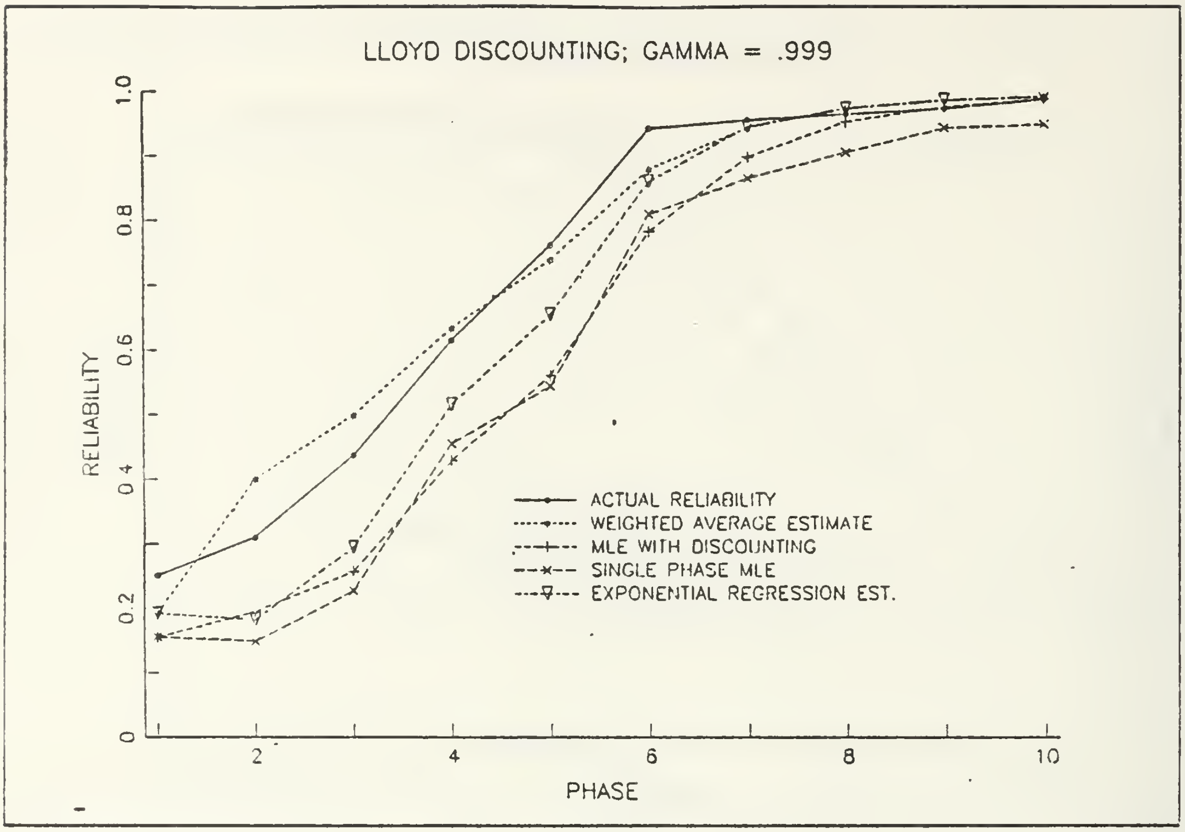

STD DISCOUNTING, N = 20 ; F = .25
RELIABILITY
PHASE
ACTUAL RELIABILITY
WEIGHTED AVERAGE ESTIMATE
MLE WITH DISCOUNTING
SINGLE PHASE MLE
EXPONENTIAL REGRESSION EST.


STD DISCOUNTING, N = 50 ; F = .25
RELIABILITY
0
0.2
0.4
0.6
0.8
1.0
2
4
6
8
10
PHASE
ACTUAL RELIABILITY
WEIGHTED AVERAGE ESTIMATE
MLE WITH DISCOUNTING
SINGLE PHASE MLE
EXPONENTIAL REGRESSION EST.


STD DISCOUNTING, N = 50 ; F = .50.
RELIABILITY
0
0.2
0.4
0.6
0.8
1.0
2
4
6
8
10
PHASE
ACTUAL RELIABILITY
WEIGHTED AVERAGE ESTIMATE
MLE WITH DISCOUNTING
SINGLE PHASE MLE
EXPONENTIAL REGRESSION EST.

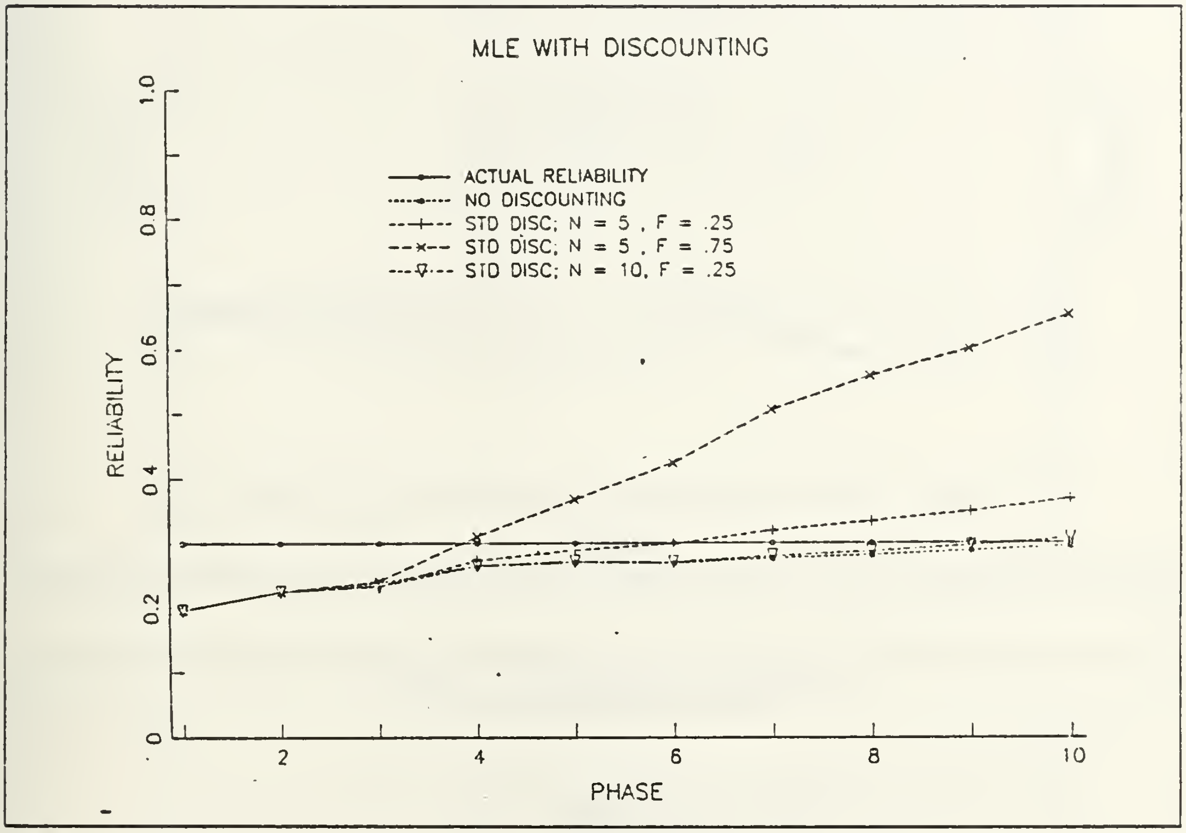

LLOYD DISCOUNTING; GAMMA = .8
RELIABILITY
0
0.2
0.4
0.6
0.8
1.0
2
4
6
8
10
PHASE
ACTUAL RELIABILITY
WEIGHTED AVERAGE ESTIMATE
MLE WITH DISCOUNTING
SINGLE PHASE MLE
EXPONENTIAL REGRESSION EST.

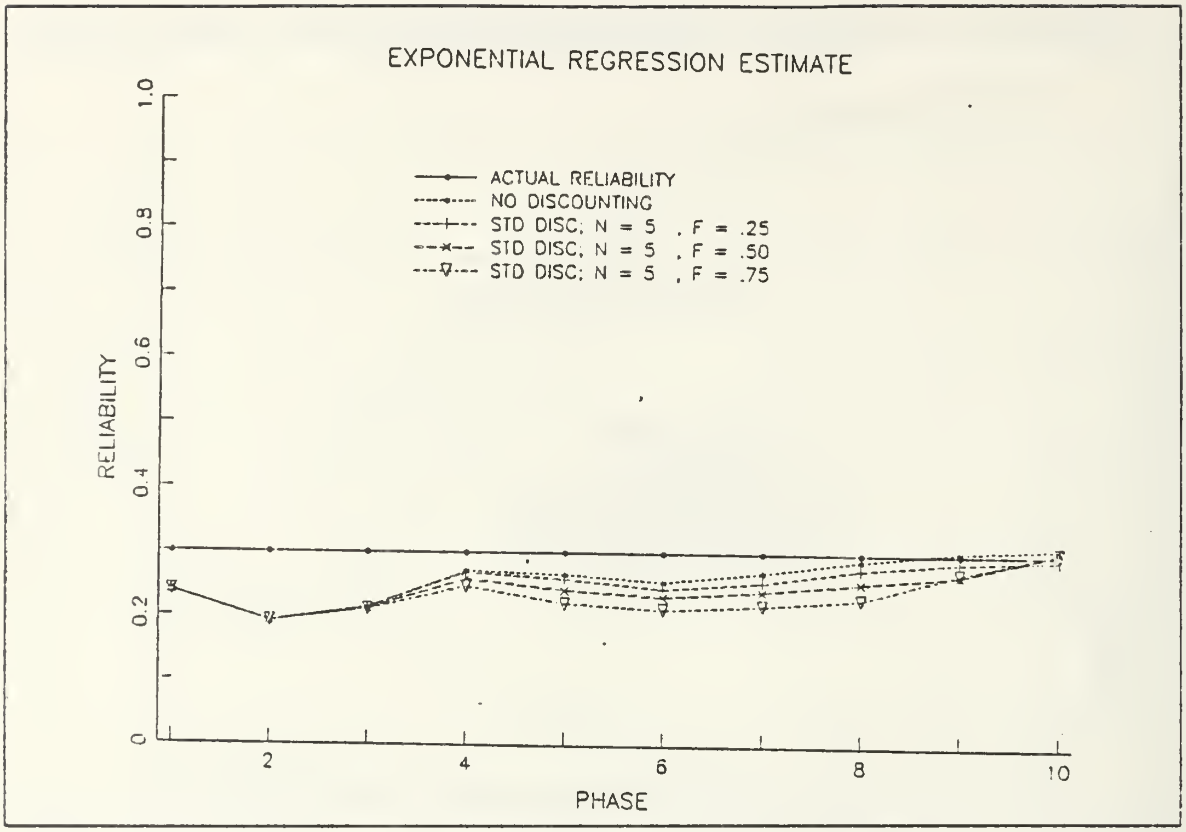

LLOYD DISCOUNTING; GAMMA = .9
RELIABILITY
0
0.2
0.4
0.6
0.8
1.0
2
4
6
8
10
PHASE
ACTUAL RELIABILITY
WEIGHTED AVERAGE ESTIMATE
MLE WITH DISCOUNTING
SINGLE PHASE MLE
EXPONENTIAL REGRESSION EST.

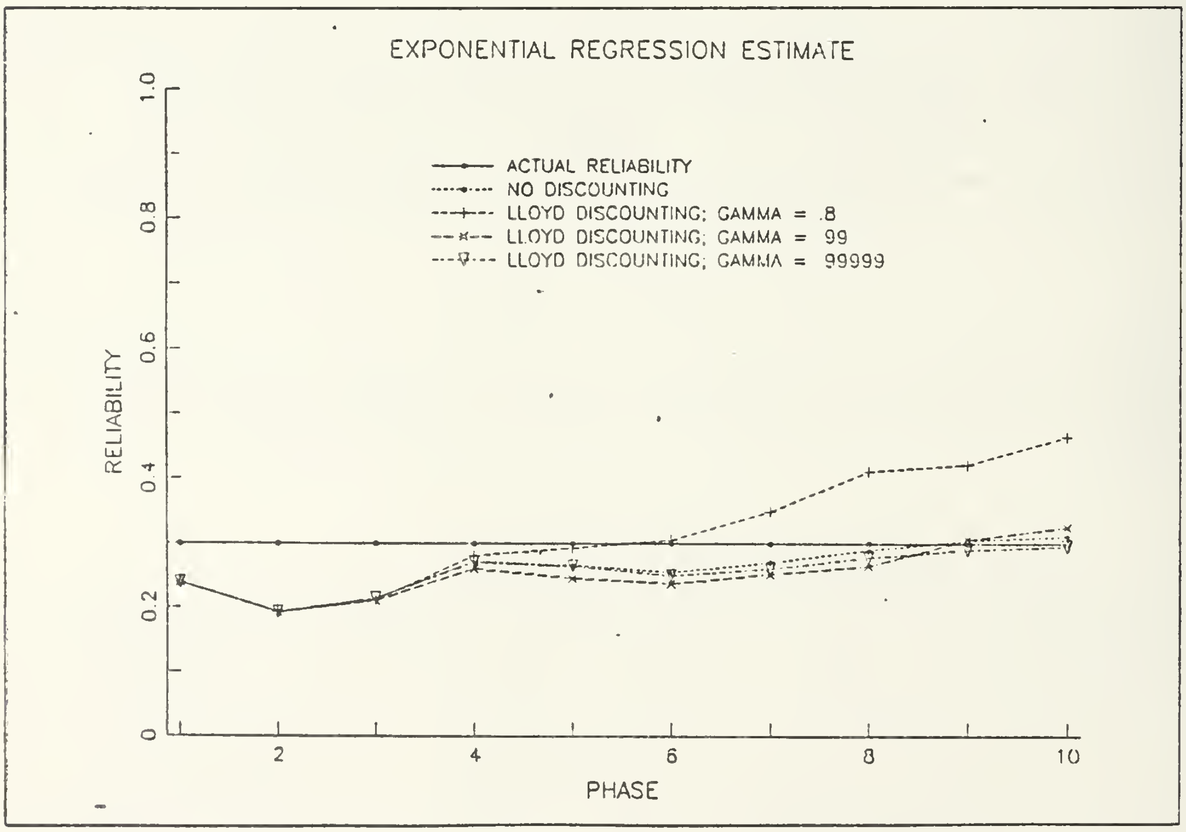


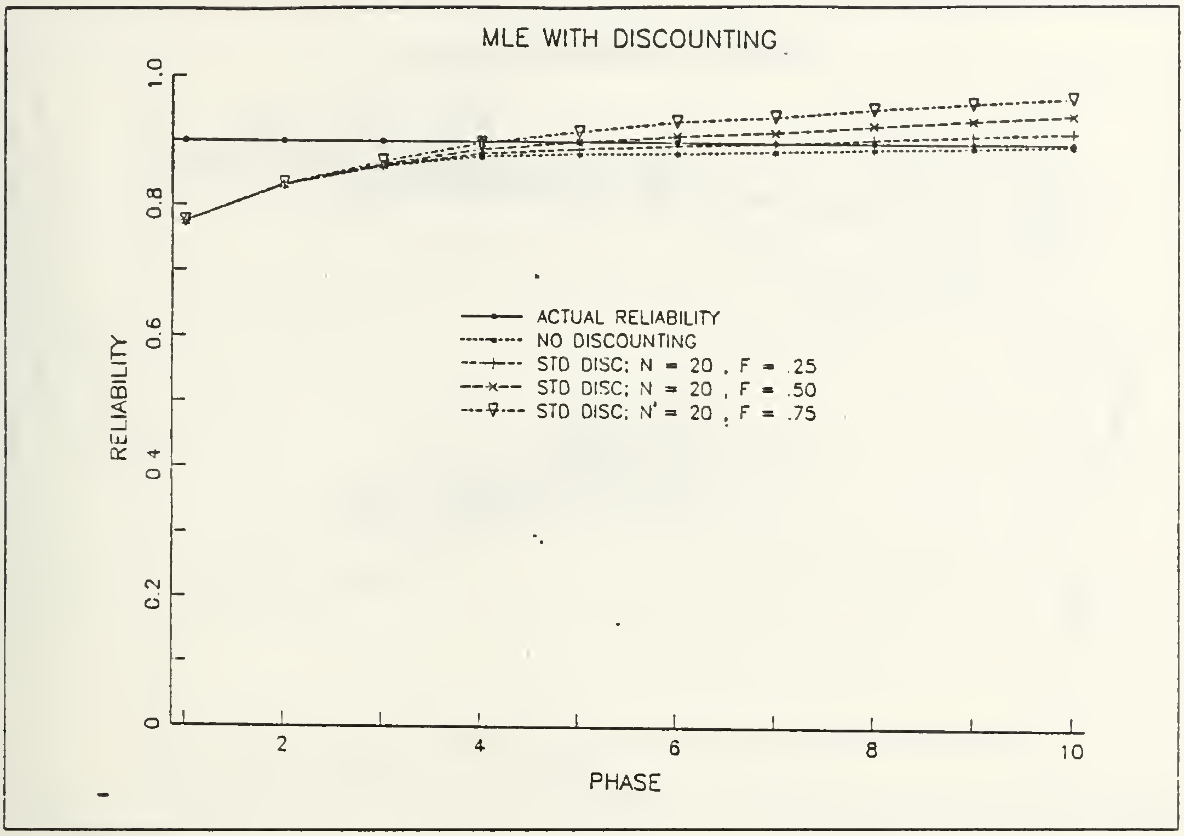
LLOYD DISCOUNTING; GAMMA = .999
RELIABILITY
0
0.2
0.4
0.6
0.8
1.0
2
4
6
8
10
PHASE
ACTUAL RELIABILITY
WEIGHTED AVERAGE ESTIMATE
MLE WITH DISCOUNTING
SINGLE PHASE MLE
EXPONENTIAL REGRESSION EST.

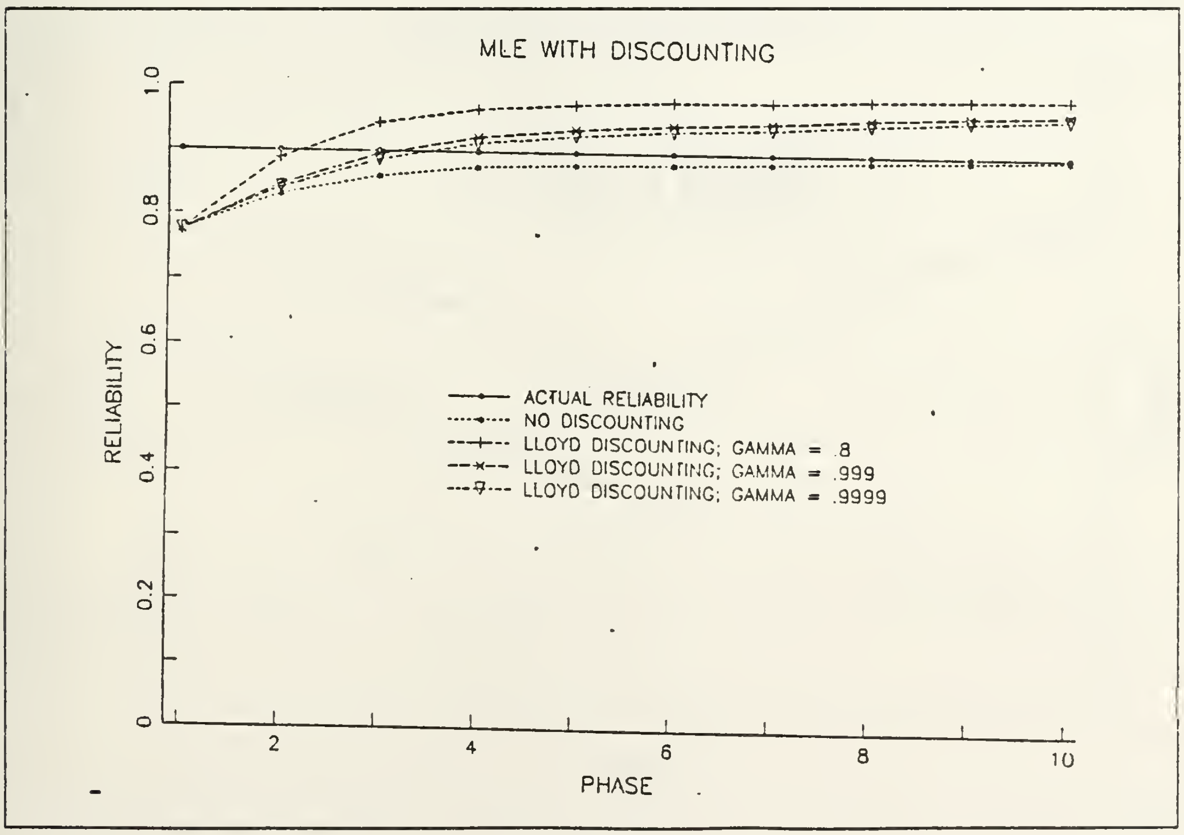
2. ESTIMATOR SENSITIVITY TO FAILURE DISCOUNTING PARAMETERS

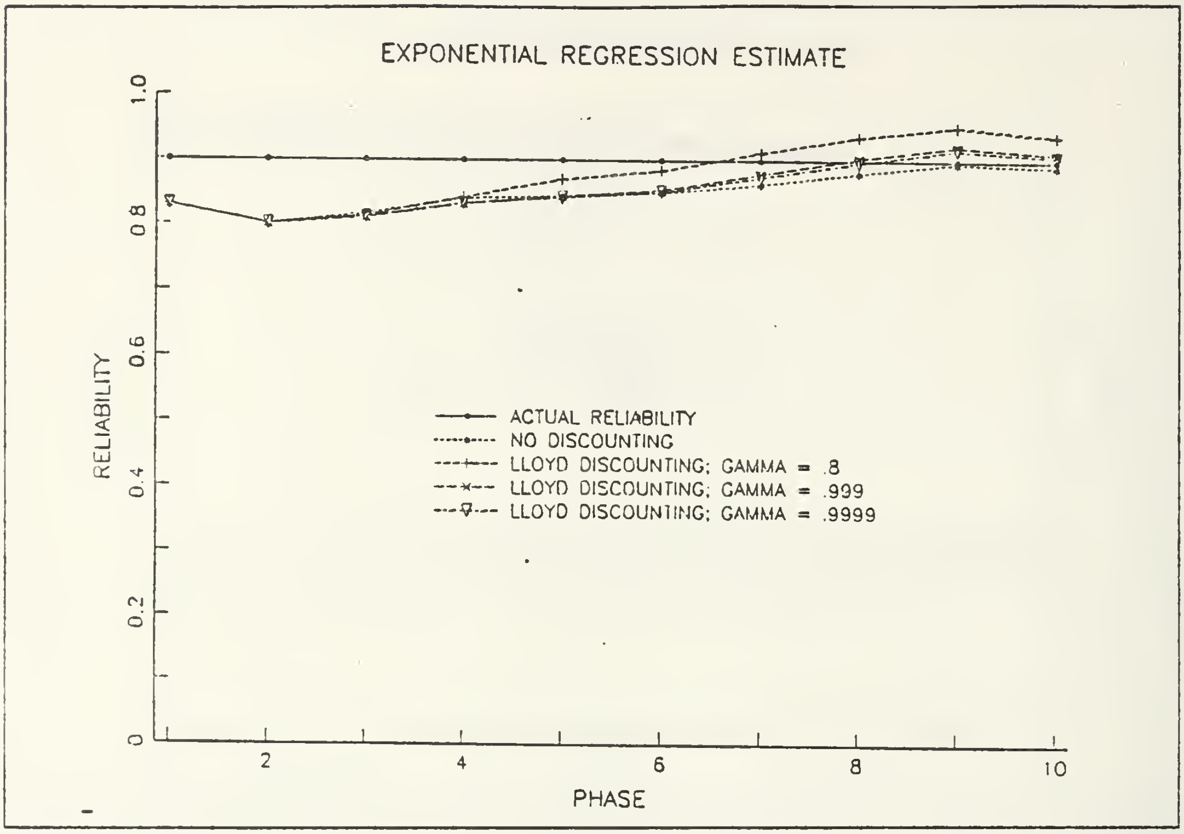


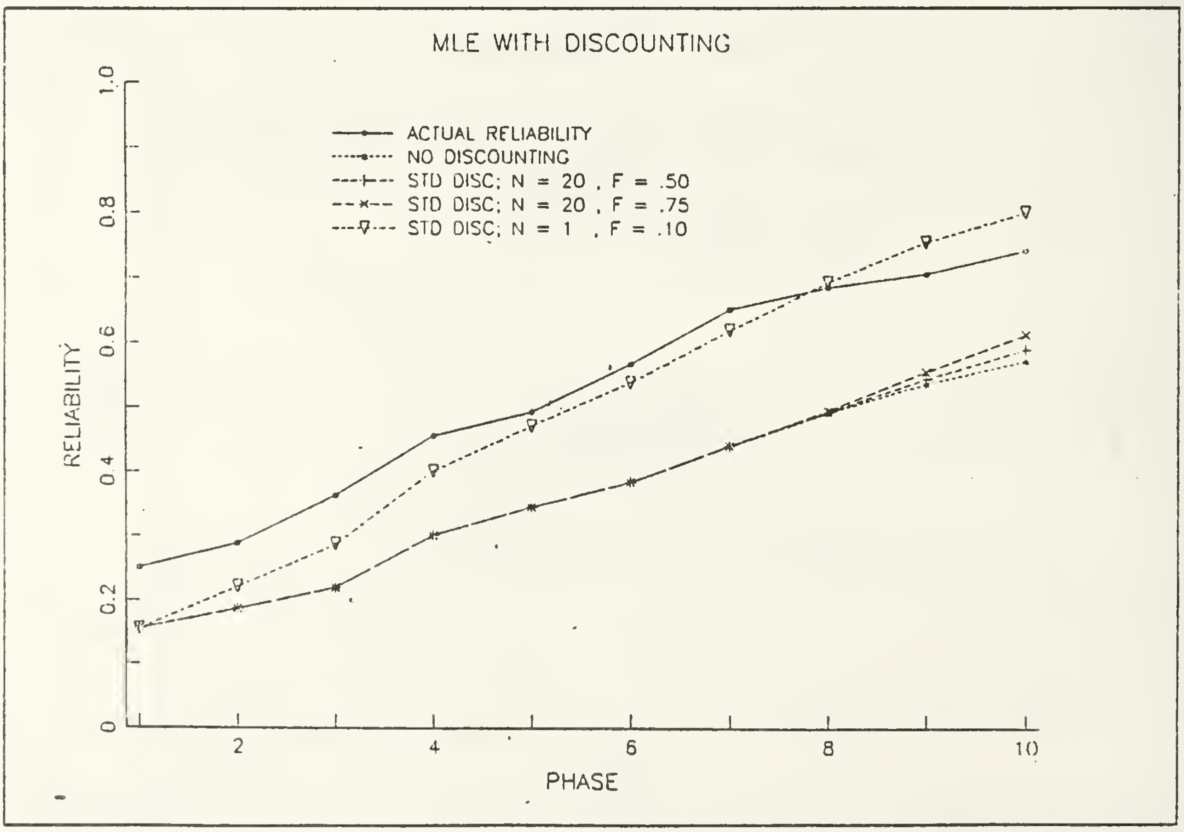

EXPONENTIAL REGRESSION ESTIMATE
ACTUAL RELIABILITY
NO DISCOUNTING
STD DISC; N = 5 , F = .25
STD DISC; N = 5 , F = .50
STD DISC; N = 5 , F = .75
RELIABILITY
0
0.2
0.4
0.6
0.8
1.0
2
4
6
8
10
PHASE

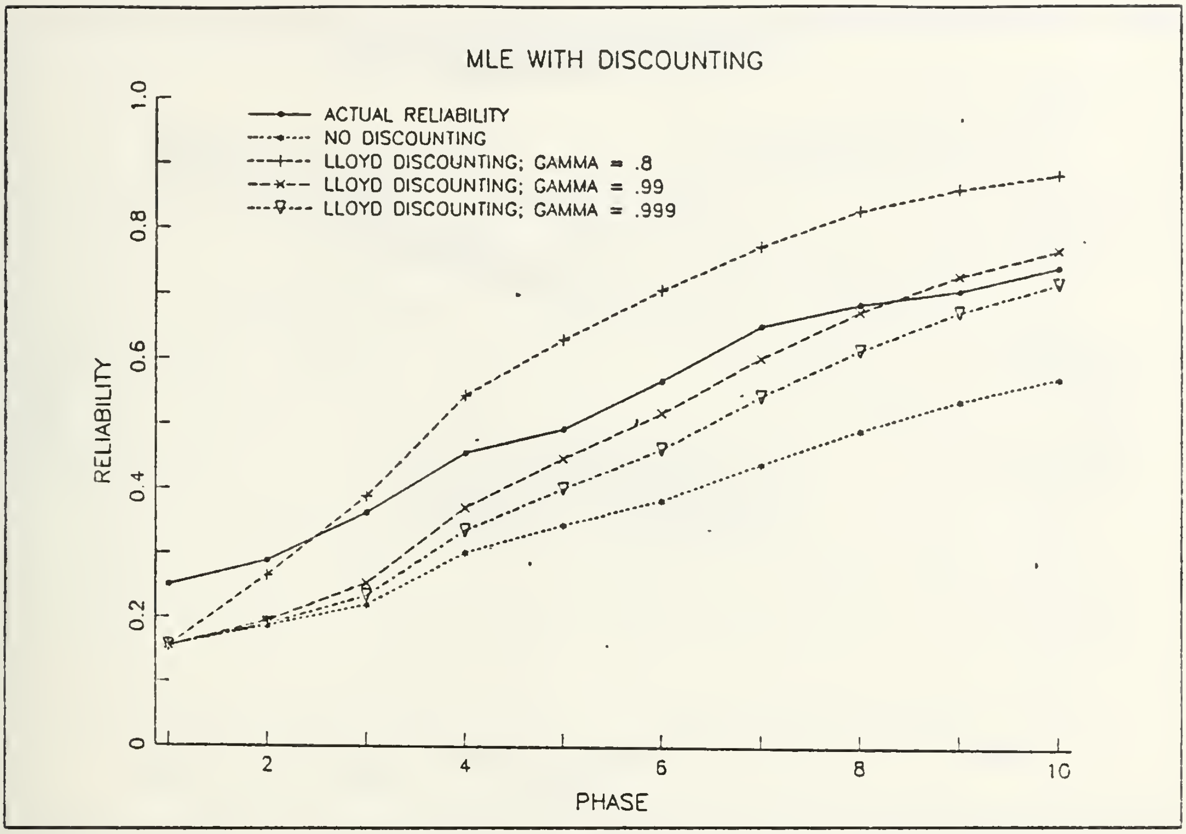

EXPONENTIAL REGRESSION ESTIMATE
ACTUAL RELIABILITY
NO DISCOUNTING
LLOYD DISCOUNTING; GAMMA = .8
LLOYD DISCOUNTING; GAMMA = .99
LLOYD DISCOUNTING; GAMMA = .99999
RELIABILITY
0
0.2
0.4
0.6
0.8
1.0
2
4
6
8
10
PHASE

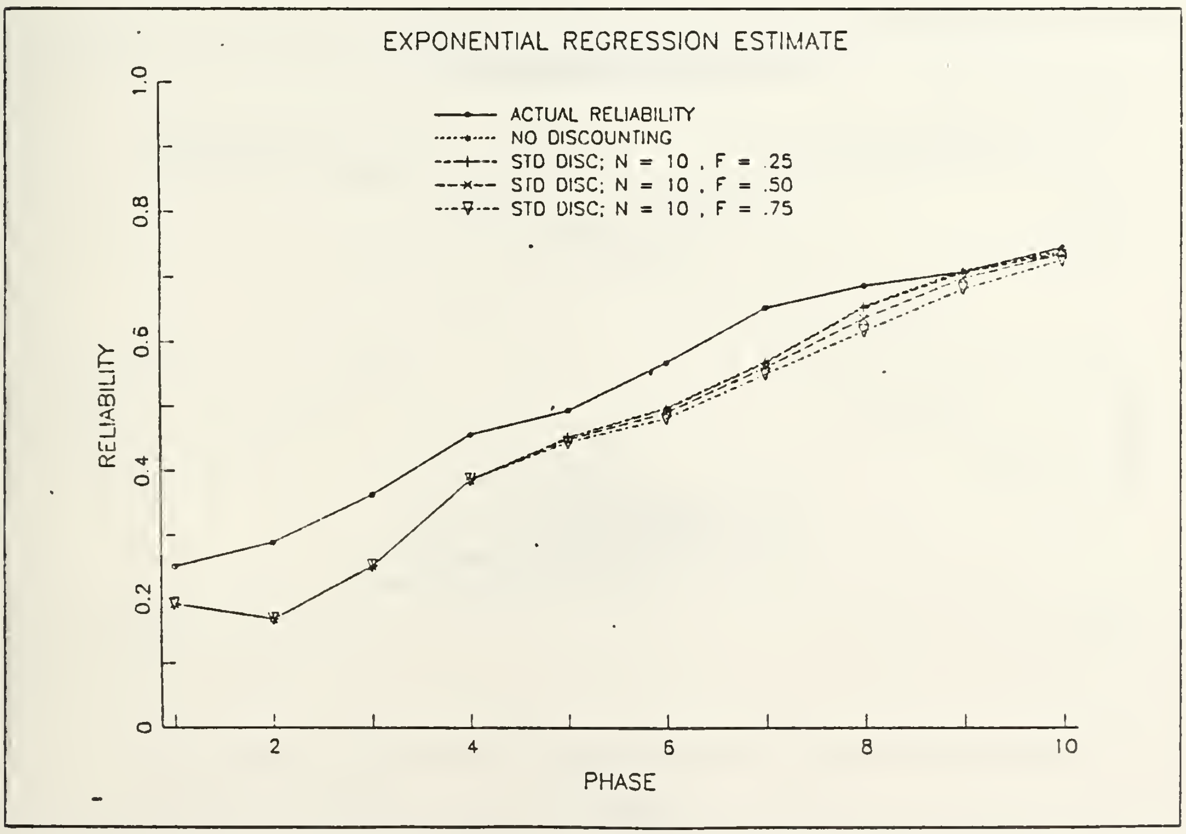

MLE WITH DISCOUNTING
RELIABILITY
PHASE
ACTUAL RELIABILITY
NO DISCOUNTING
STD DISC: N = 20 , F = .25
STD DISC: N = 20 , F = .50
STD DISC: N' = 20 , F = .75
0
0.2
0.4
0.6
0.8
1.0
2
4
6
8
10

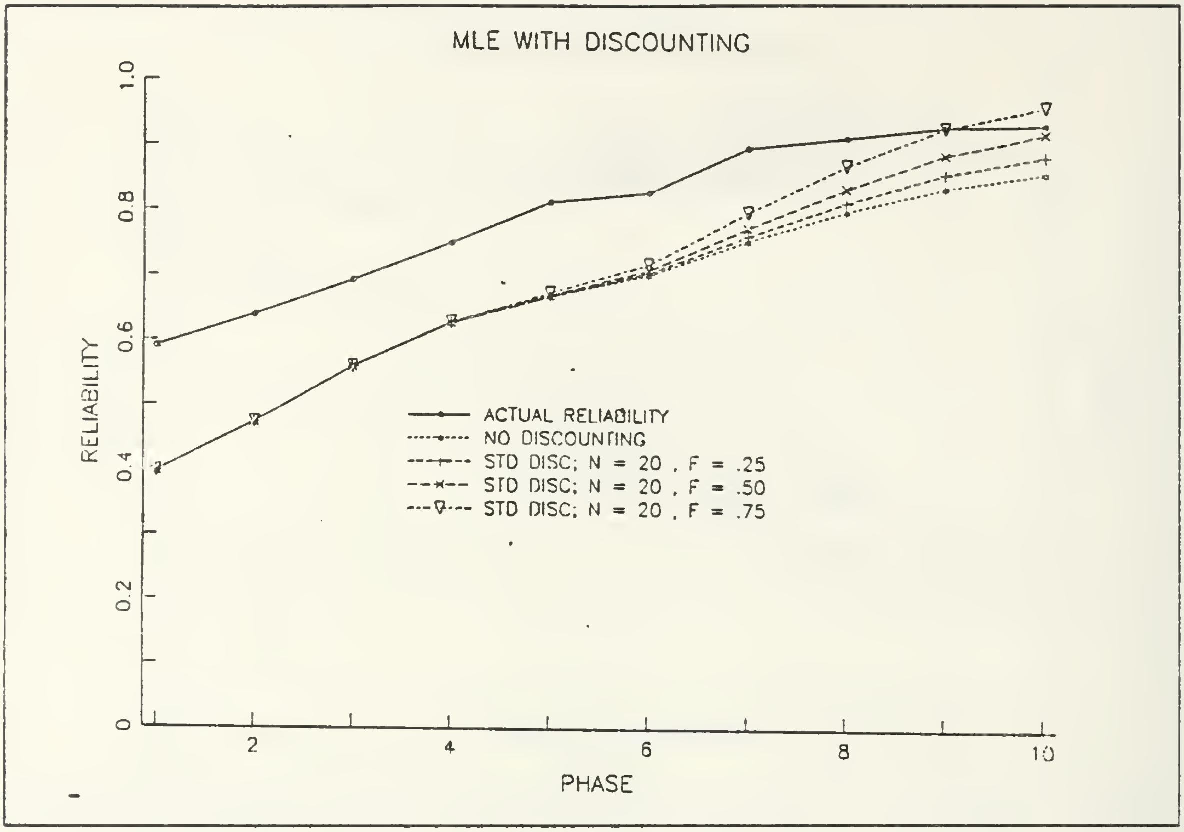

MLE WITH DISCOUNTING
RELIABILITY
PHASE
ACTUAL RELIABILITY
NO DISCOUNTING
LLOYD DISCOUNTING; GAMMA = .8
LLOYD DISCOUNTING; GAMMA = .999
LLOYD DISCOUNTING; GAMMA = .9999
0
0.2
0.4
0.6
0.8
1.0
2
4
6
8
10

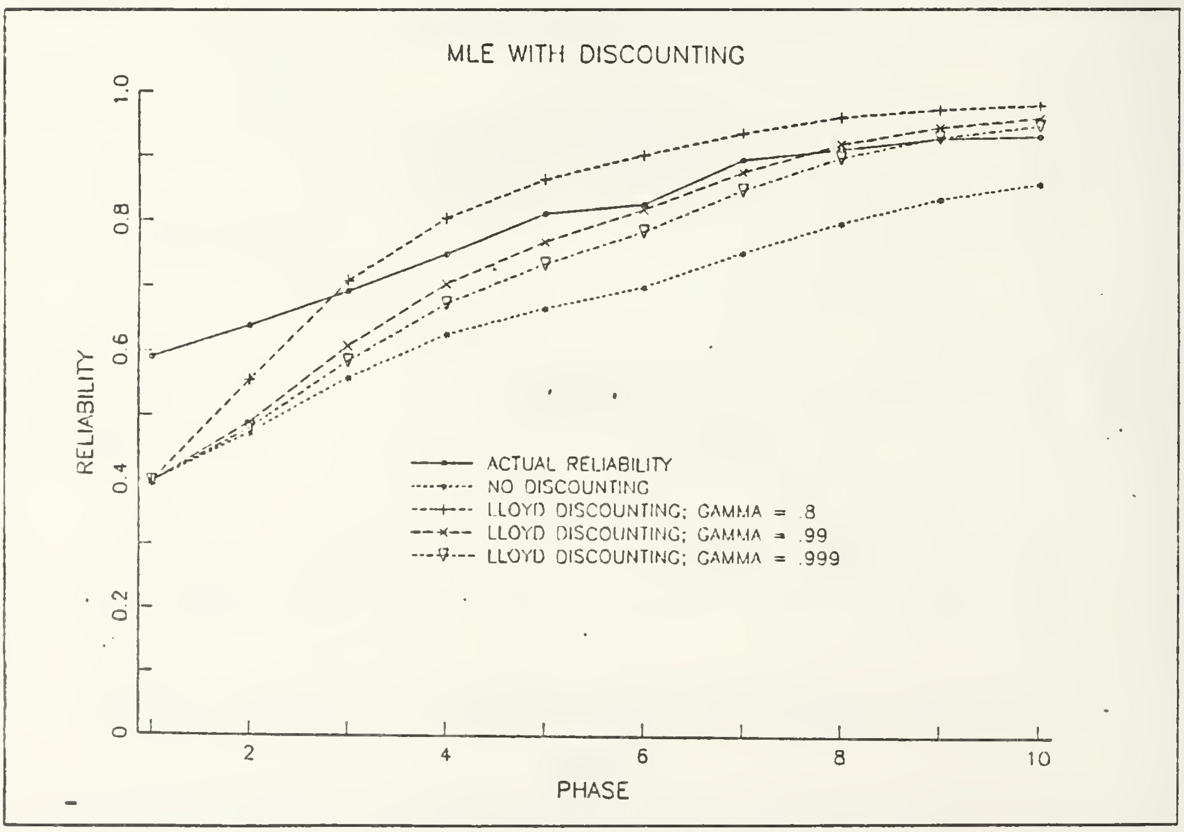

EXPONENTIAL REGRESSION ESTIMATE
RELIABILITY
0
0.2
0.4
0.6
0.8
1.0
2
4
6
8
10
PHASE
ACTUAL RELIABILITY
NO DISCOUNTING
LLOYD DISCOUNTING; GAMMA = .8
LLOYD DISCOUNTING; GAMMA = .999
LLOYD DISCOUNTING; GAMMA = .9999

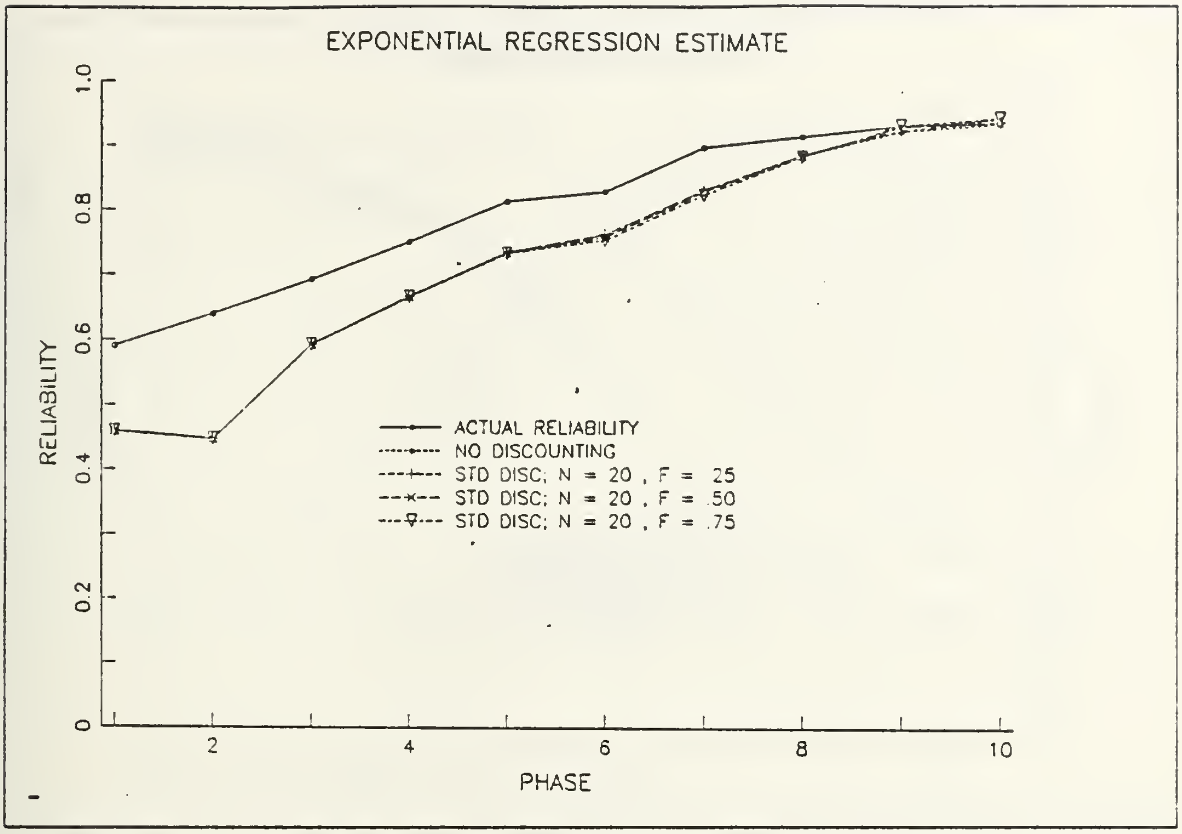

MLE WITH DISCOUNTING
RELIABILITY
0
0.2
0.4
0.6
0.8
1.0
2
4
6
8
10
PHASE
ACTUAL RELIABILITY
NO DISCOUNTING
STD DISC; N = 20 , F = .50
STD DISC; N = 20 , F = .75
STD DISC; N = 1 , F = .10

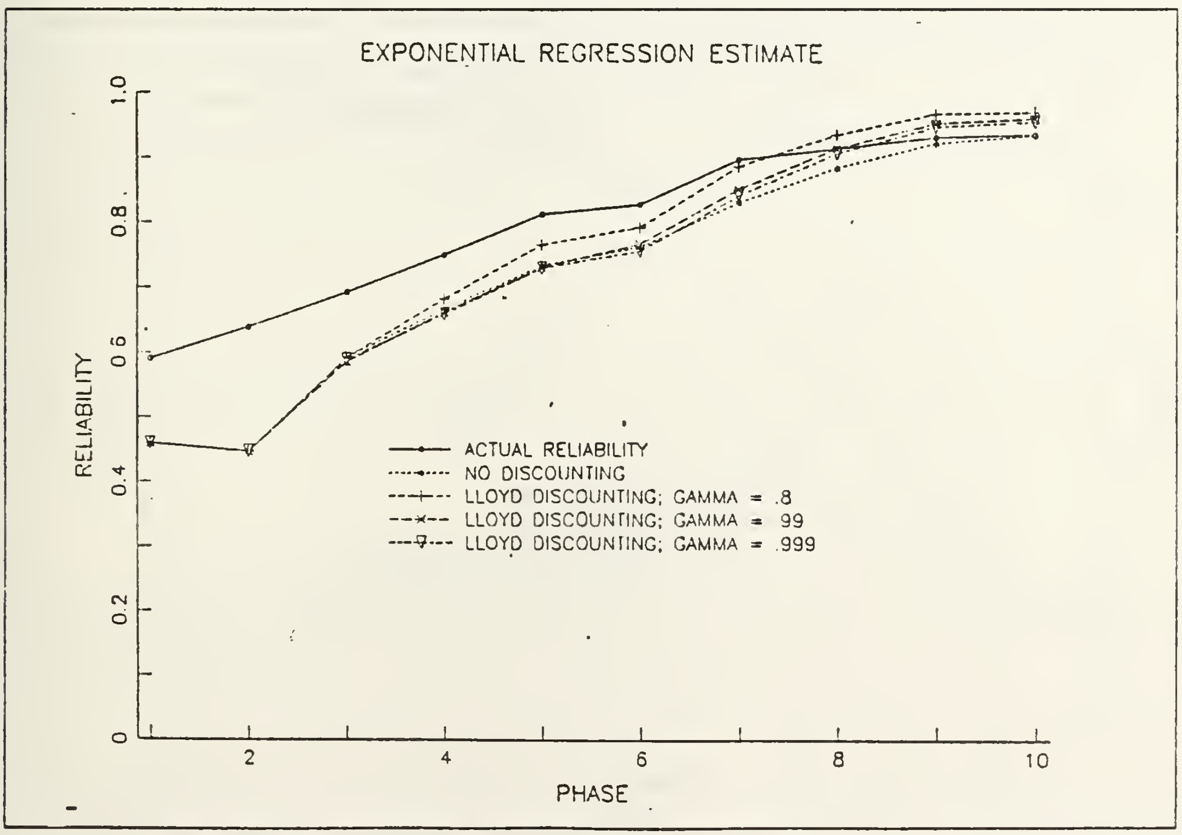

MLE WITH DISCOUNTING
ACTUAL RELIABILITY
NO DISCOUNTING
LLOYD DISCOUNTING; GAMMA = .8
LLOYD DISCOUNTING; GAMMA = .99
LLOYD DISCOUNTING; GAMMA = .999
RELIABILITY
0
0.2
0.4
0.6
0.8
1.0
2
4
6
8
10
PHASE

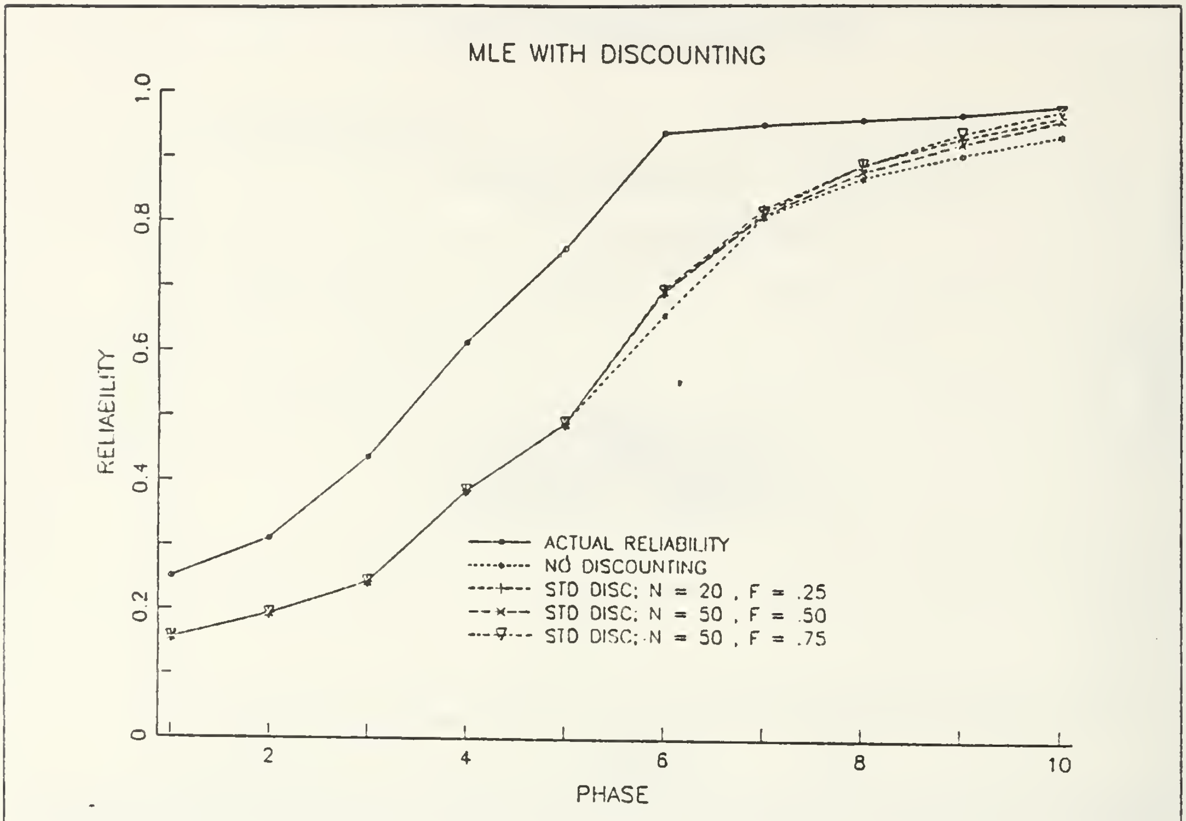

EXPONENTIAL REGRESSION ESTIMATE
ACTUAL RELIABILITY
NO DISCOUNTING
STD DISC; N = 10 , F = .25
STD DISC; N = 10 , F = .50
STD DISC; N = 10 , F = .75
RELIABILITY
0
0.2
0.4
0.6
0.8
1.0
2
4
6
8
10
PHASE

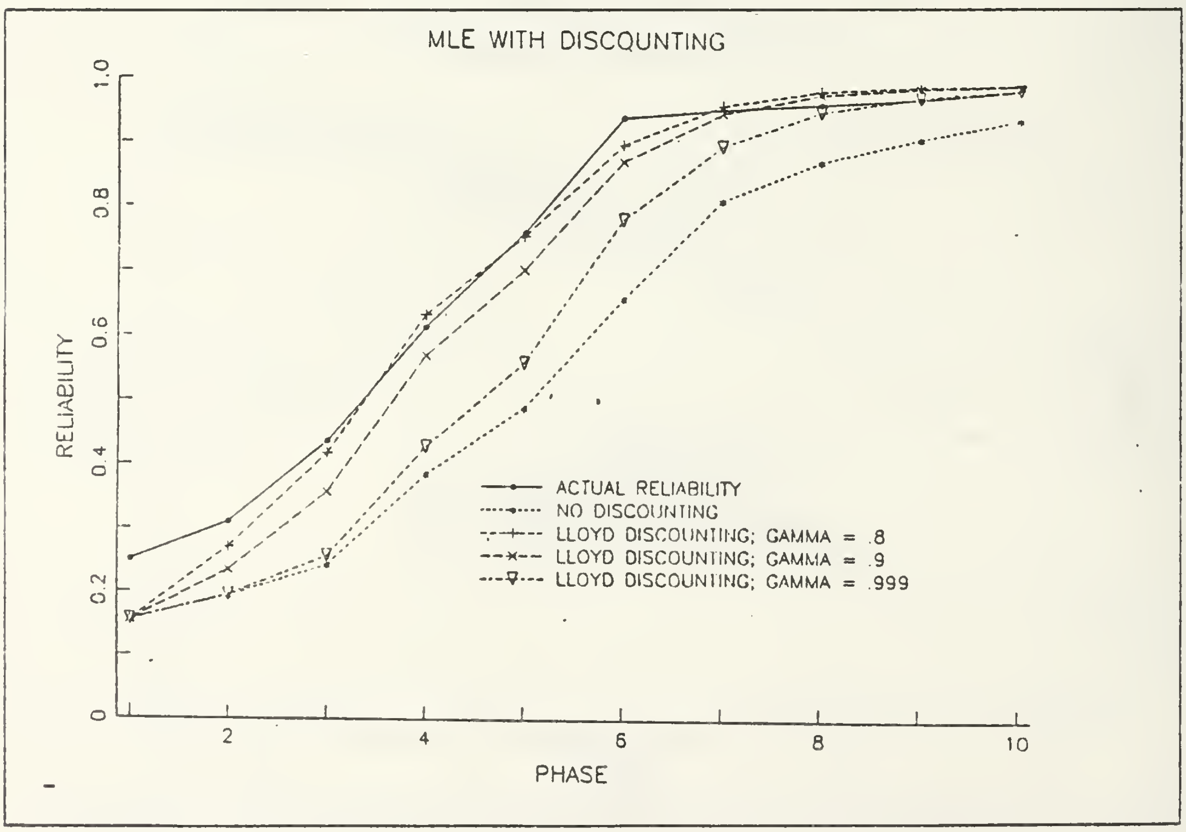

MLE WITH DISCOUNTING
RELIABILITY
0
0.2
0.4
0.6
0.8
1.0
2
4
6
8
10
PHASE
ACTUAL RELIABILITY
NO DISCOUNTING
STD DISC; N = 20 , F = .25
STD DISC; N = 20 , F = .50
STD DISC; N = 20 , F = .75

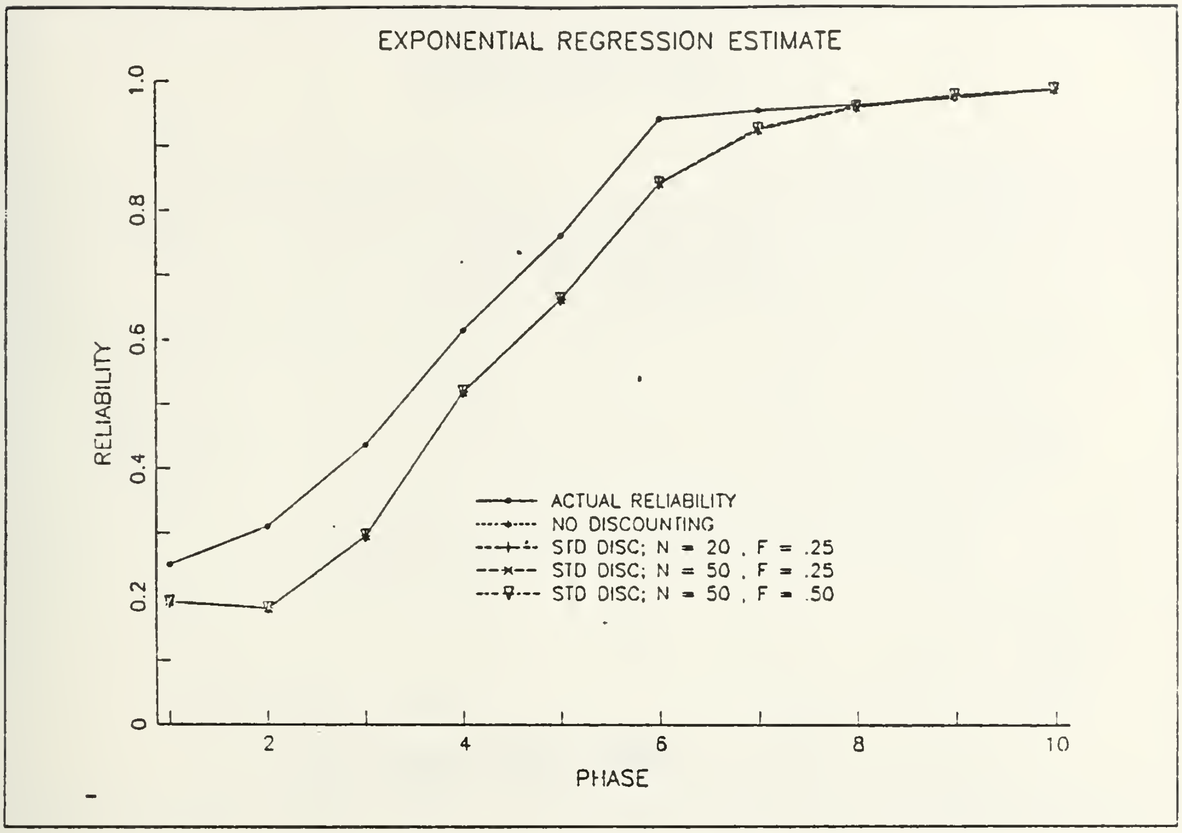

MLE WITH DISCOUNTING
RELIABILITY
0
0.2
0.4
0.6
0.8
1.0
2
4
6
8
10
PHASE
ACTUAL RELIABILITY
NO DISCOUNTING
LLOYD DISCOUNTING; GAMMA = .8
LLOYD DISCOUNTING; GAMMA = .99
LLOYD DISCOUNTING; GAMMA = .999

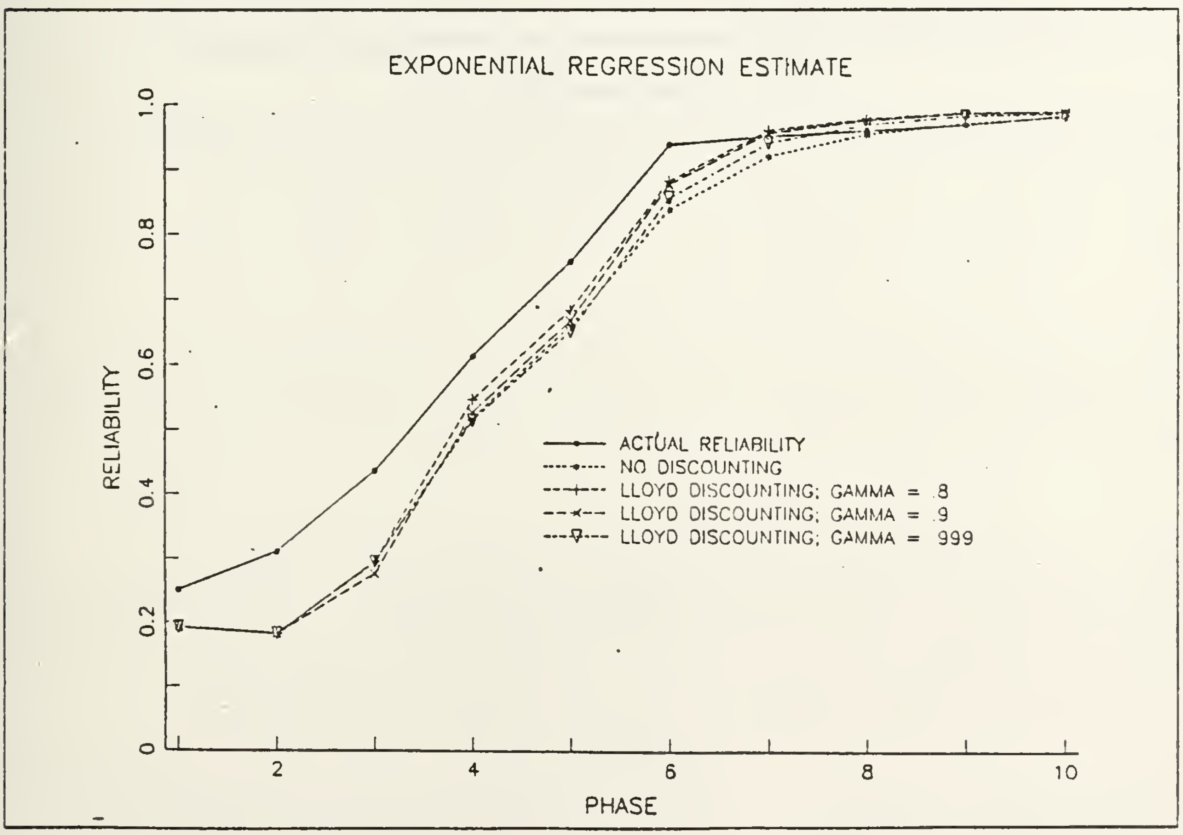

EXPONENTIAL REGRESSION ESTIMATE
RELIABILITY
0
0.2
0.4
0.6
0.8
1.0
2
4
6
8
10
PHASE
ACTUAL RELIABILITY
NO DISCOUNTING
STD DISC; N = 20 , F = .25
STD DISC; N = 20 , F = .50
STD DISC; N = 20 , F = .75


EXPONENTIAL REGRESSION ESTIMATE
RELIABILITY
0
0.2
0.4
0.6
0.8
1.0
2
4
6
8
10
PHASE
ACTUAL RELIABILITY
NO DISCOUNTING
LLOYD DISCOUNTING; GAMMA = .8
LLOYD DISCOUNTING; GAMMA = .99
LLOYD DISCOUNTING; GAMMA = .999

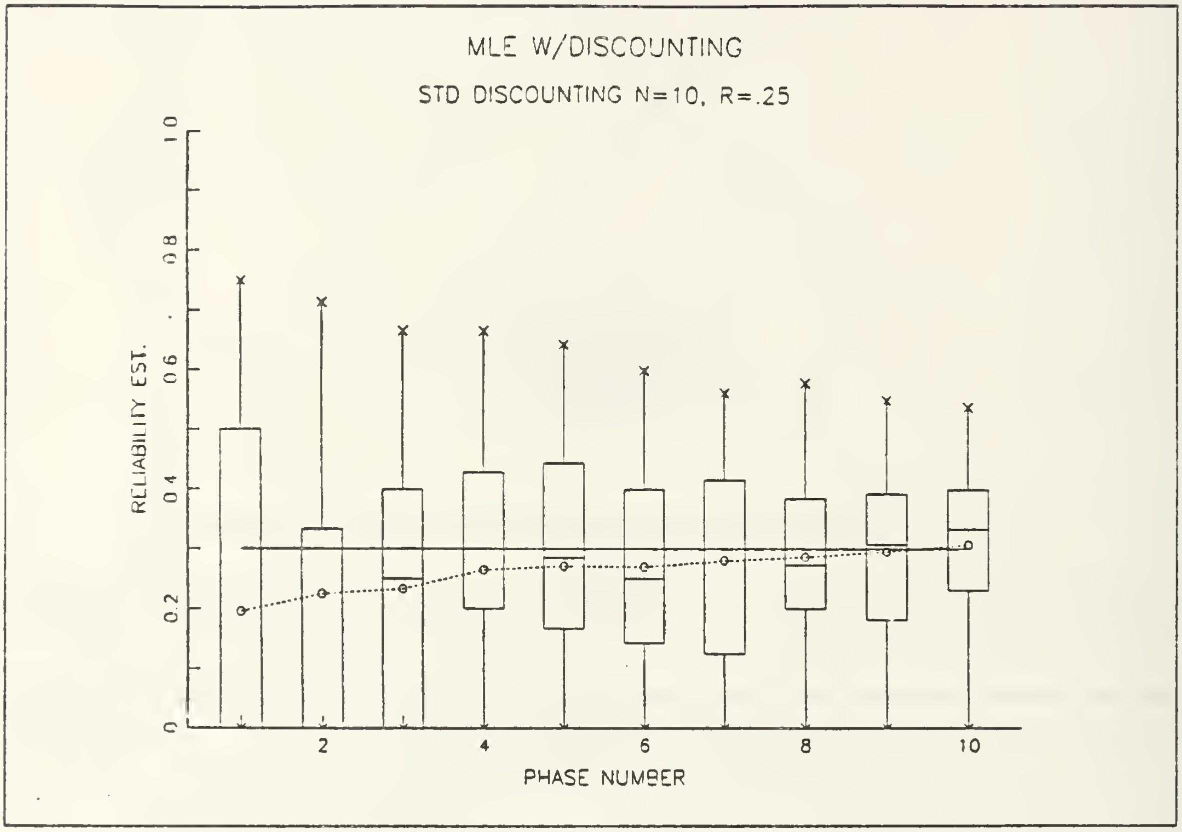

MLE WITH DISCOUNTING
RELIABILITY
PHASE
0
0.2
0.4
0.6
0.8
1.0
2
4
6
8
10
ACTUAL RELIABILITY
NO DISCOUNTING
STD DISC; N = 20 , F = .25
STD DISC; N = 50 , F = .50
STD DISC; N = 50 , F = .75

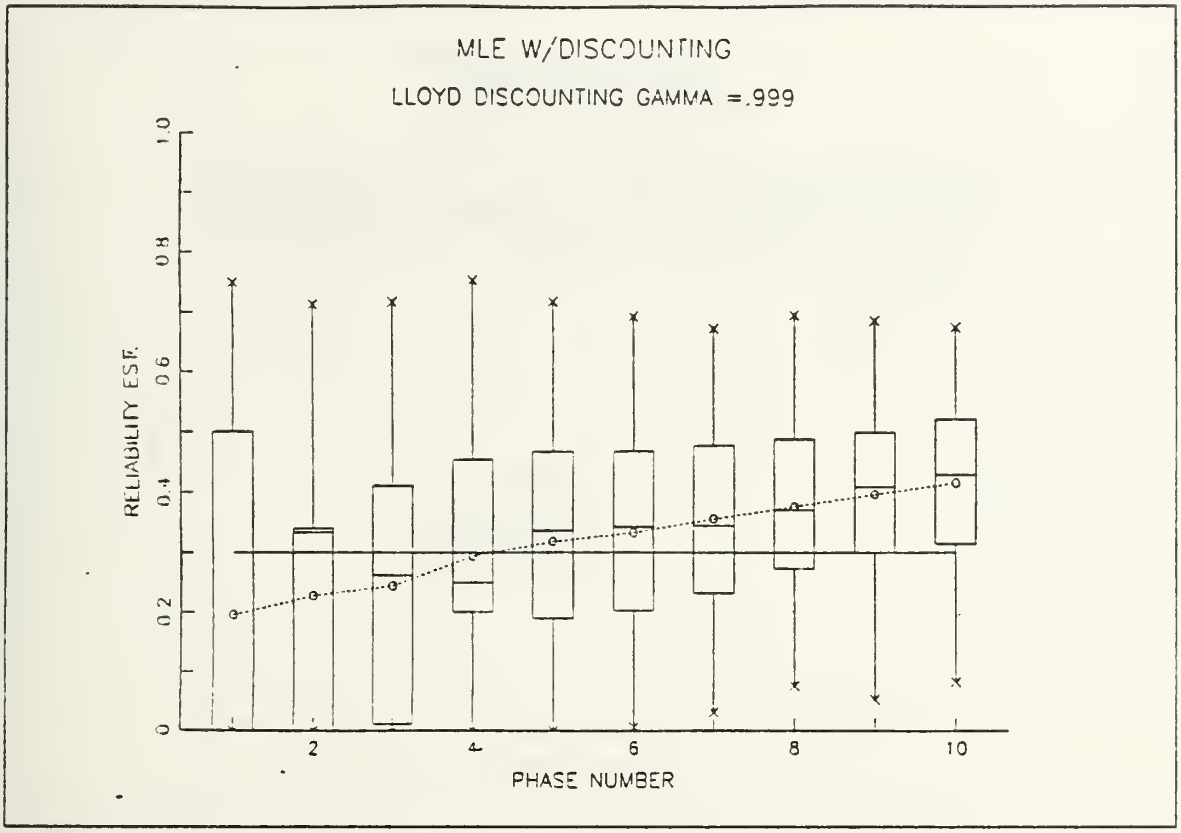

MLE WITH DISCOUNTING
RELIABILITY
PHASE
0
0.2
0.4
0.6
0.8
1.0
2
4
6
8
10
ACTUAL RELIABILITY
NO DISCOUNTING
LLOYD DISCOUNTING; GAMMA = .8
LLOYD DISCOUNTING; GAMMA = .9
LLOYD DISCOUNTING; GAMMA = .999

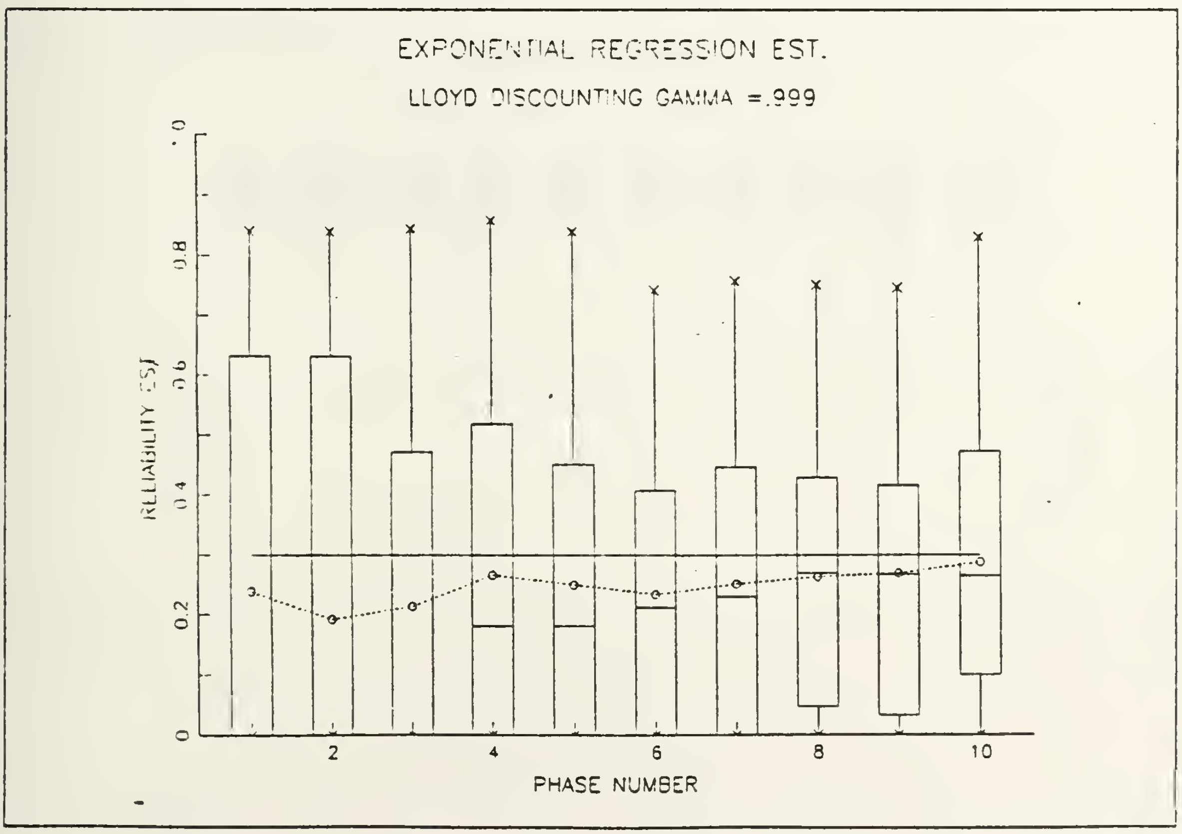

EXPONENTIAL REGRESSION ESTIMATE
RELIABILITY
0
0.2
0.4
0.6
0.8
1.0
2
4
6
8
10
PHASE
ACTUAL RELIABILITY
NO DISCOUNTING
STD DISC; N = 20 , F = .25
STD DISC; N = 50 , F = .25
STD DISC; N = 50 , F = .50

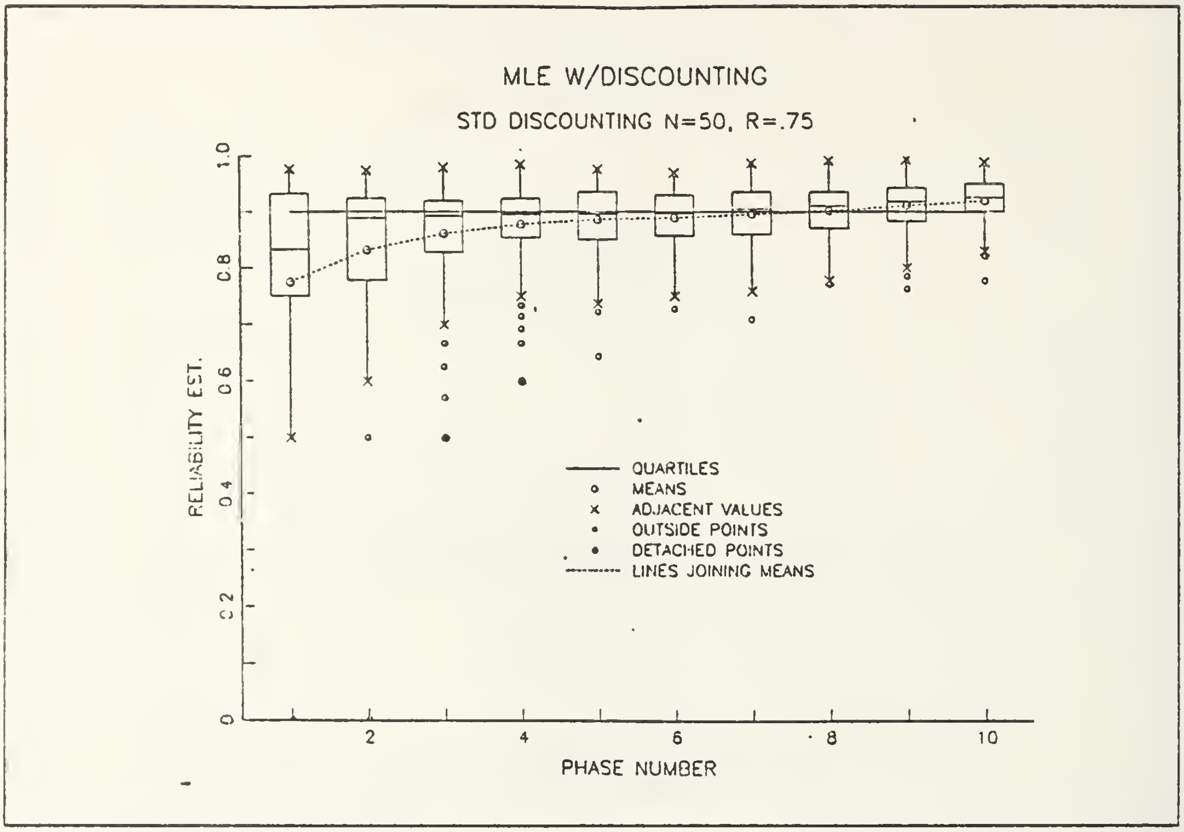

EXPONENTIAL REGRESSION ESTIMATE
RELIABILITY
0
0.2
0.4
0.6
0.8
1.0
2
4
6
8
10
PHASE
ACTUAL RELIABILITY
NO DISCOUNTING
LLOYD DISCOUNTING; GAMMA = .8
LLOYD DISCOUNTING; GAMMA = .9
LLOYD DISCOUNTING; GAMMA = .999

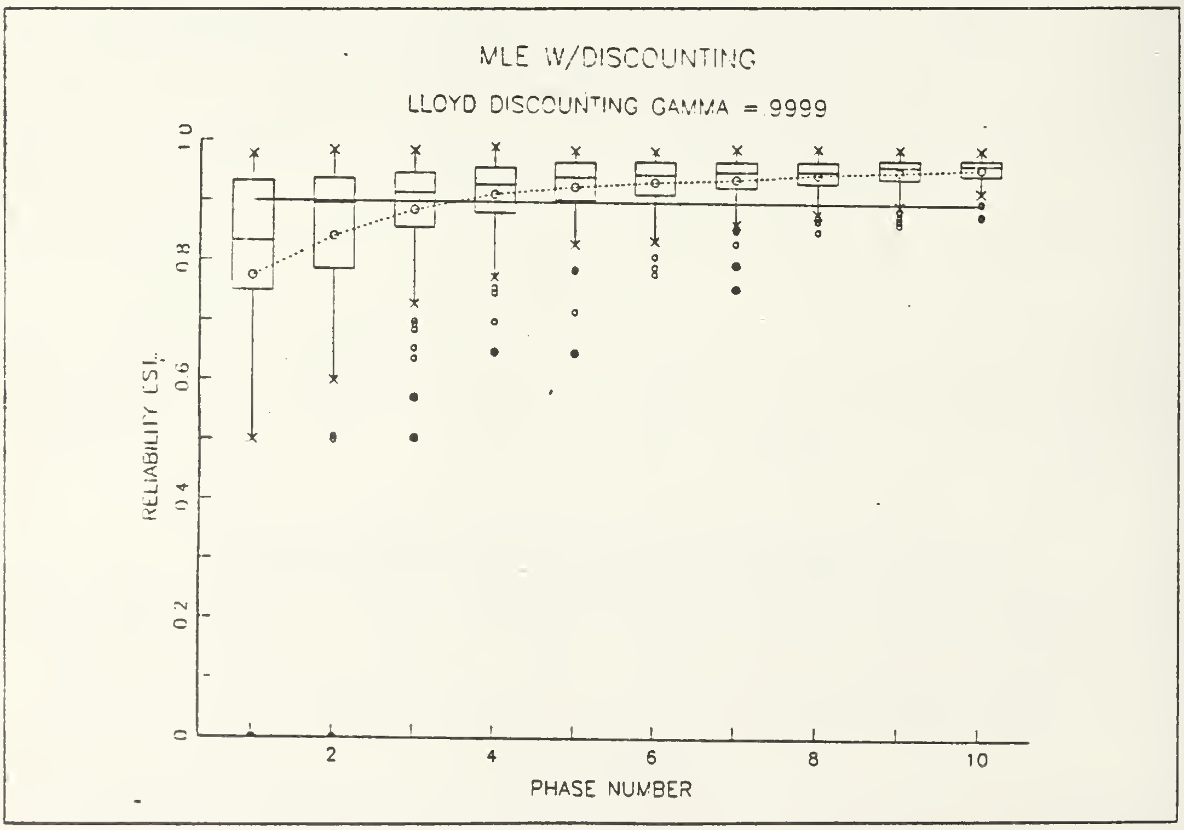
3. ESTIMATOR VARIABILITY (MULTIPLE BOX PLOTS)

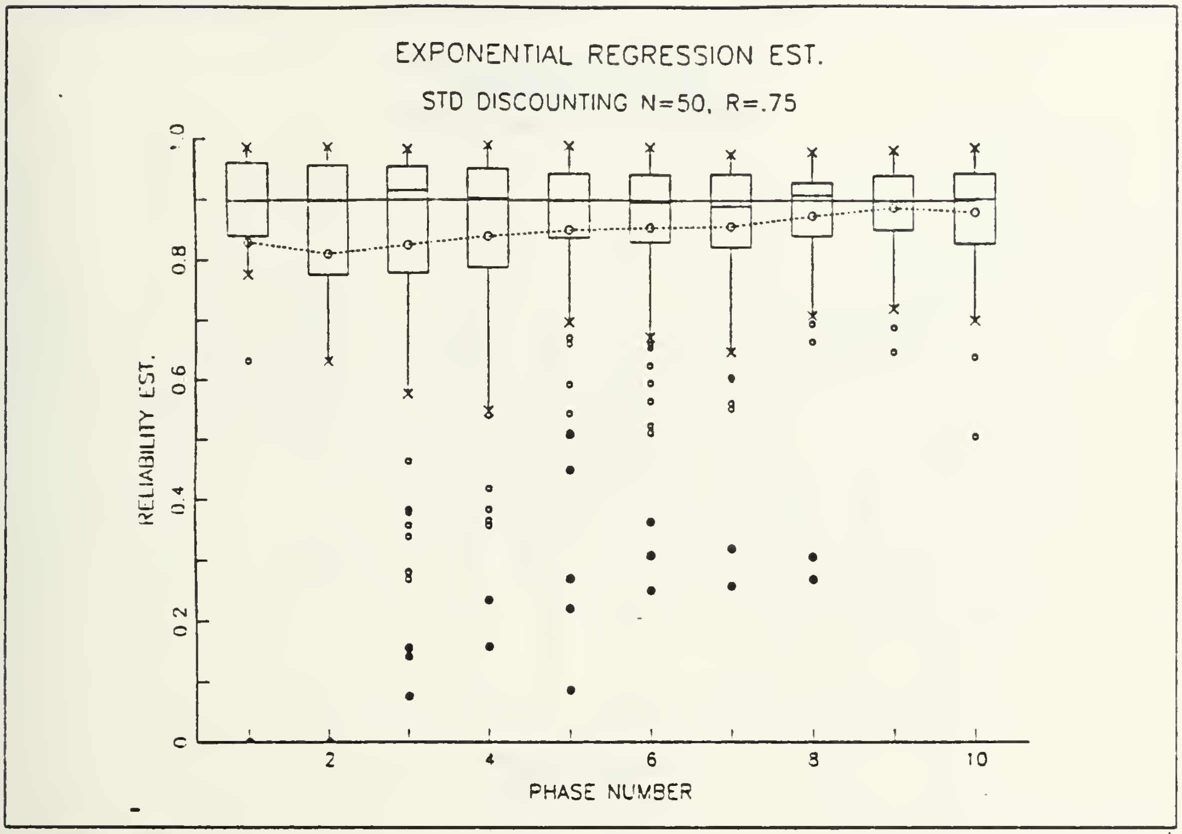



MLE W/DISCOUNTING
LLOYD DISCOUNTING GAMMA =.999
RELIABILITY EST.
0
0.2
0.4
0.6
0.8
1.0
2
4
6
8
10
PHASE NUMBER

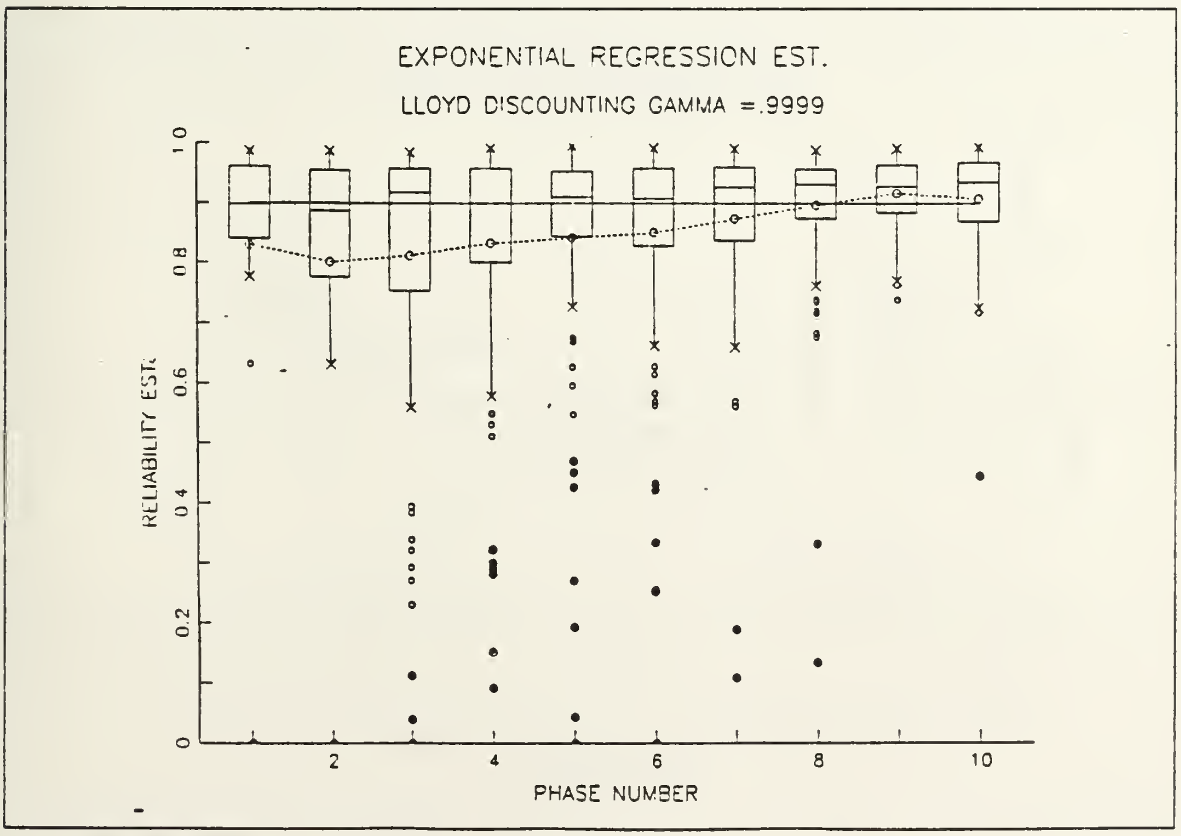

EXPONENTIAL REGRESSION EST.
LLOYD DISCOUNTING GAMMA =.999
0
0.2
0.8
2
4
6
8
10
PHASE NUMBER


MLE W/DISCOUNTING
STD DISCOUNTING N=50, R=.75
RELIABILITY EST.
PHASE NUMBER
QUARTILES
MEANS
ADJACENT VALUES
OUTSIDE POINTS
DETACHED POINTS
LINES JOINING MEANS

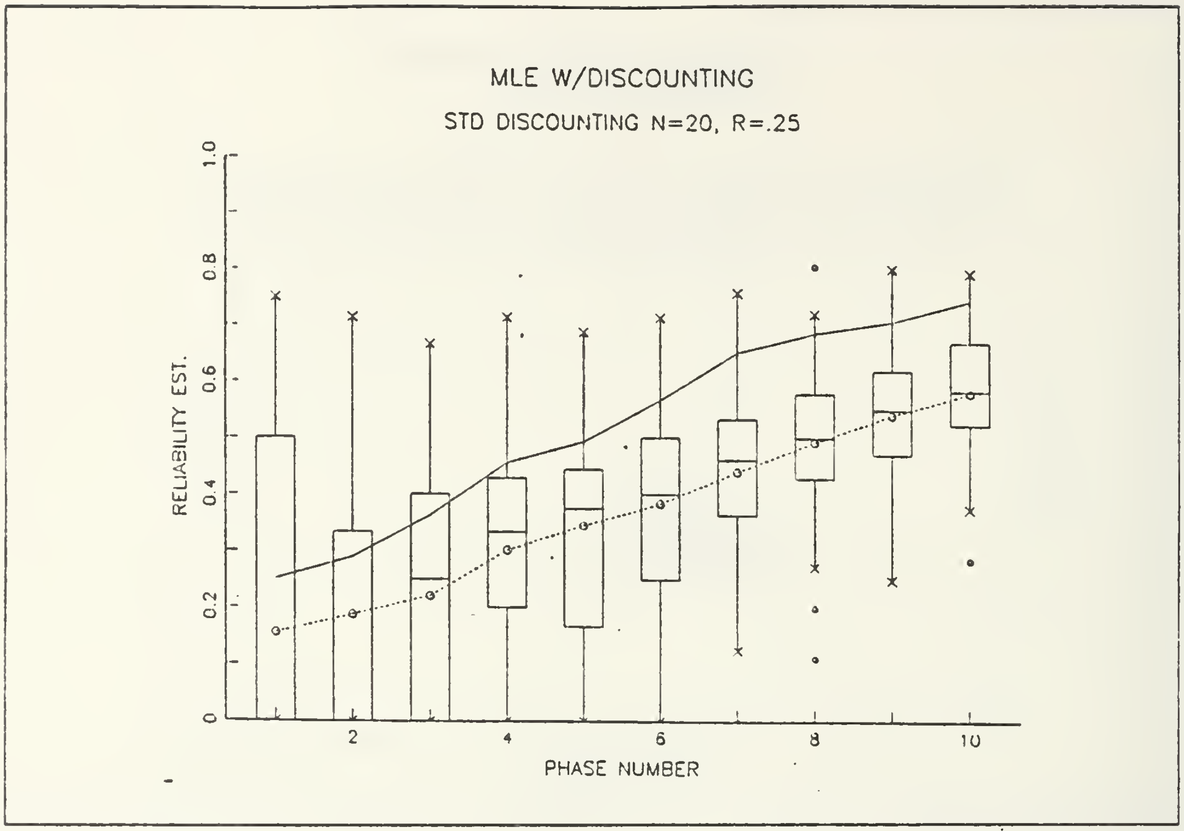

MLE W/DISCOUNTING
LLOYD DISCOUNTING GAMMA = .9999
RELIABILITY EST.
PHASE NUMBER

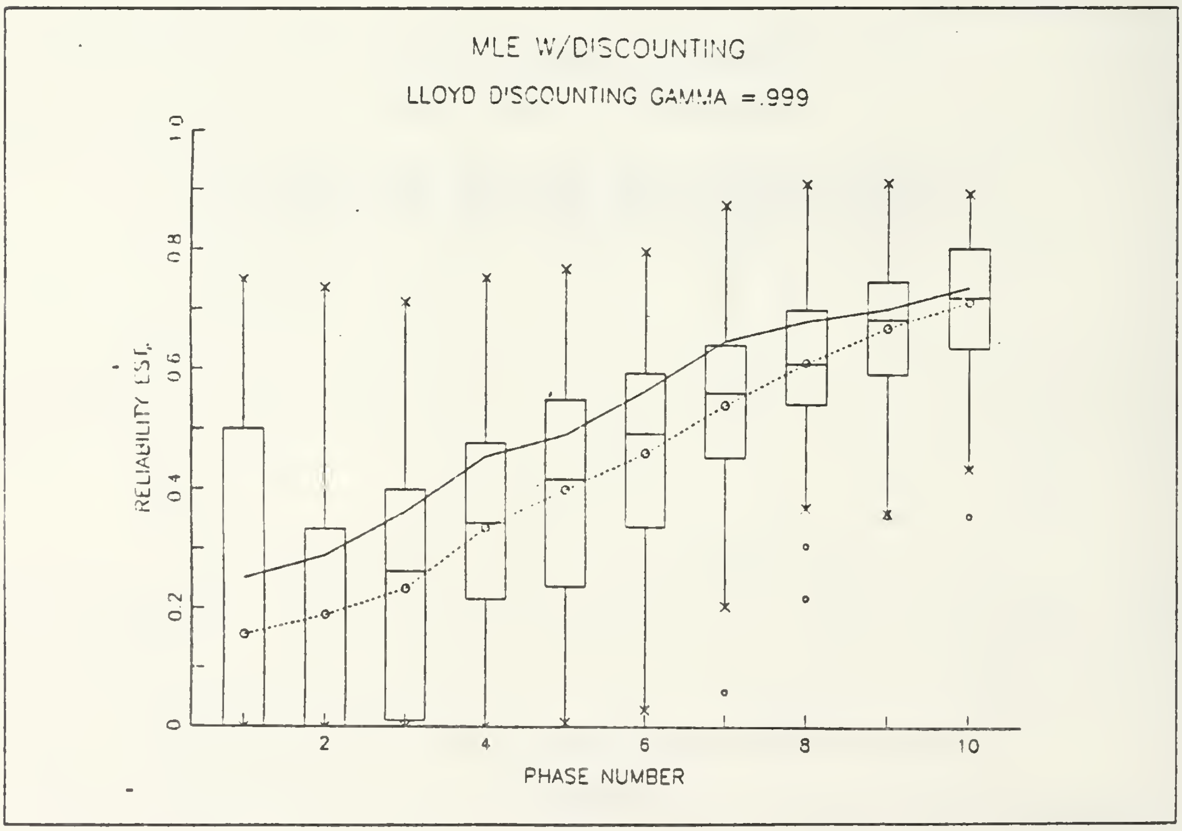


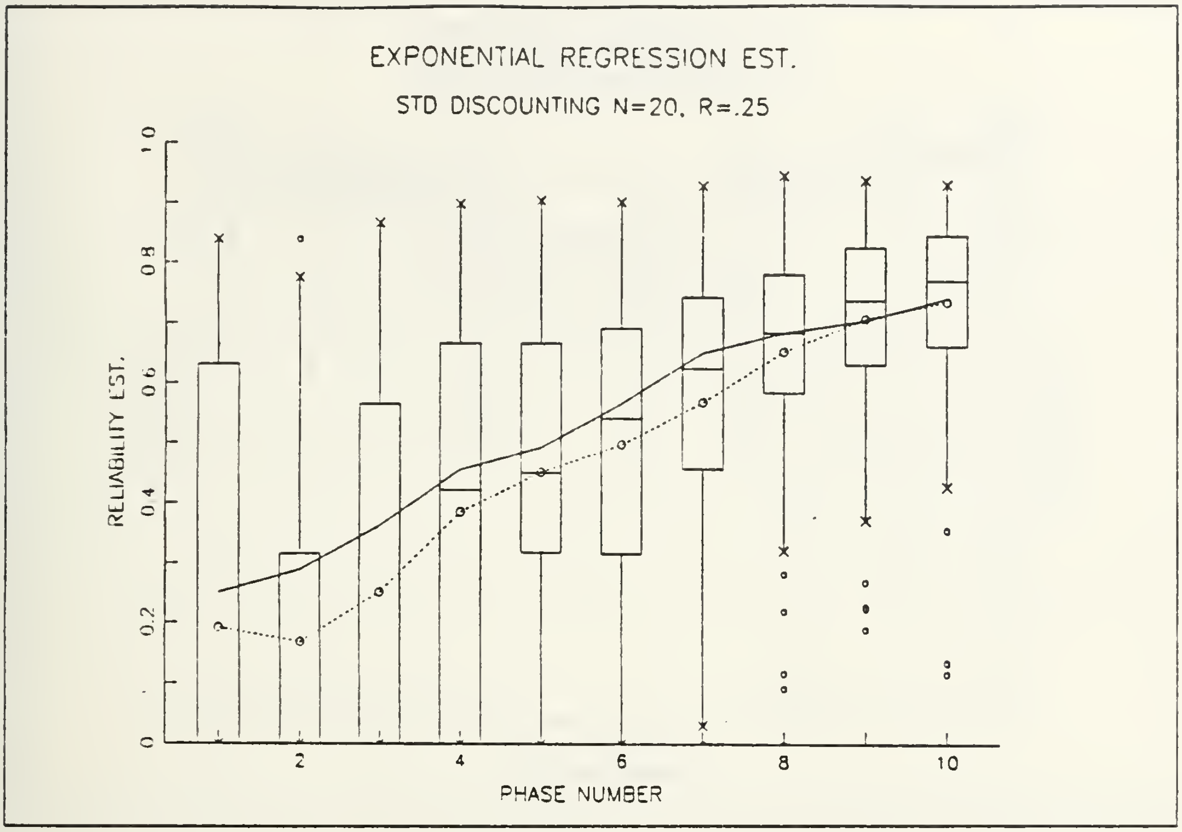
EXPONENTIAL REGRESSION EST.
STD DISCOUNTING N=50, R=.75
RELIABILITY EST.
0
0.2
0.4
0.6
0.8
1.0
2
4
6
8
10
PHASE NUMBER



EXPONENTIAL REGRESSION EST.
LLOYD DISCOUNTING GAMMA =.9999
RELIABILITY EST.
0
0.2
0.4
0.6
0.8
1.0
2
4
6
8
10
PHASE NUMBER


MLE W/DISCOUNTING
STD DISCOUNTING N=20, R=.25
RELIABILITY EST.
0
0.2
0.4
0.6
0.8
1.0
2
4
6
8
10
PHASE NUMBER


MLE W/DISCOUNTING
LLOYD DISCOUNTING GAMMA =.999
RELIABILITY EST.
0
0.2
0.4
0.6
0.8
1.0
2
4
6
8
10
PHASE NUMBER


EXPONENTIAL REGRESSION EST.
STD DISCOUNTING N=20, R=.25
RELIABILITY EST.
0
0.2
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0.6
0.8
1.0
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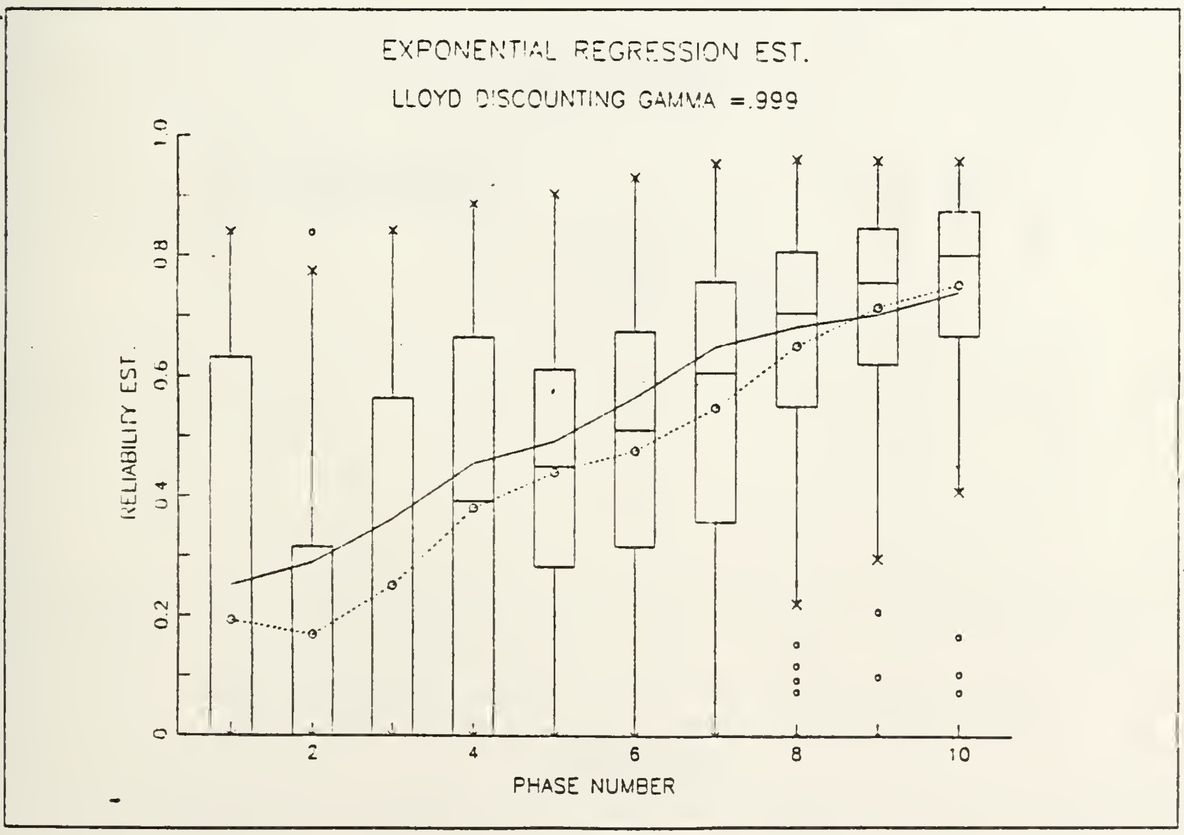
6
8
10
PHASE NUMBER


EXPONENTIAL REGRESSION EST.
LLOYD DISCOUNTING GAMMA =.999
RELIABILITY EST.
0
0.2
0.4
0.6
0.8
1.0
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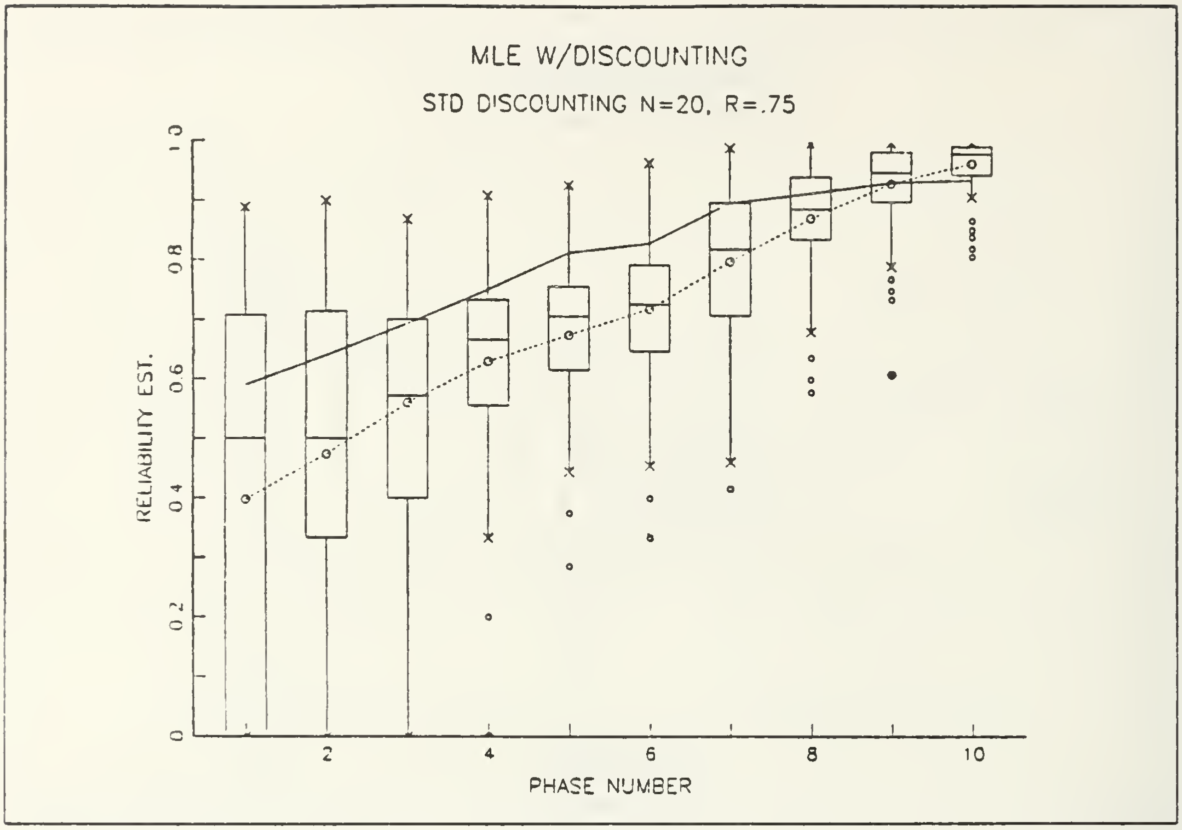
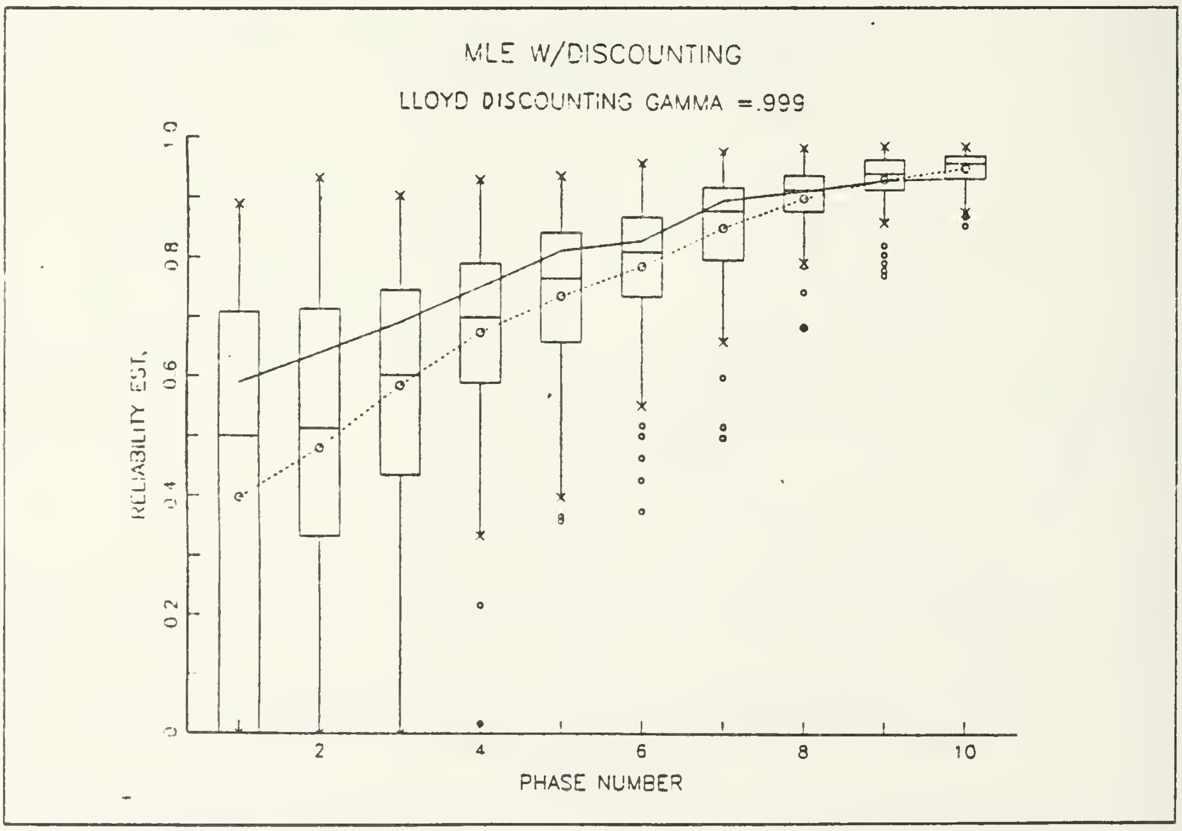
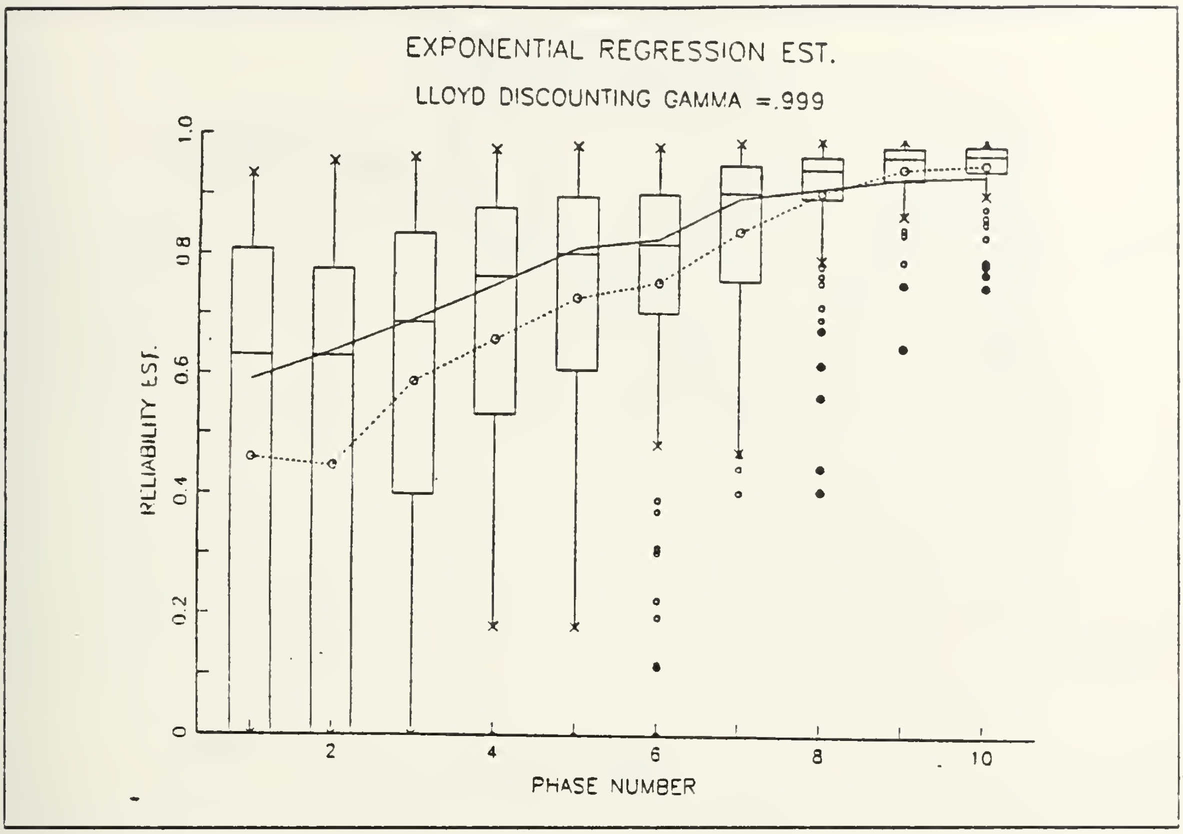
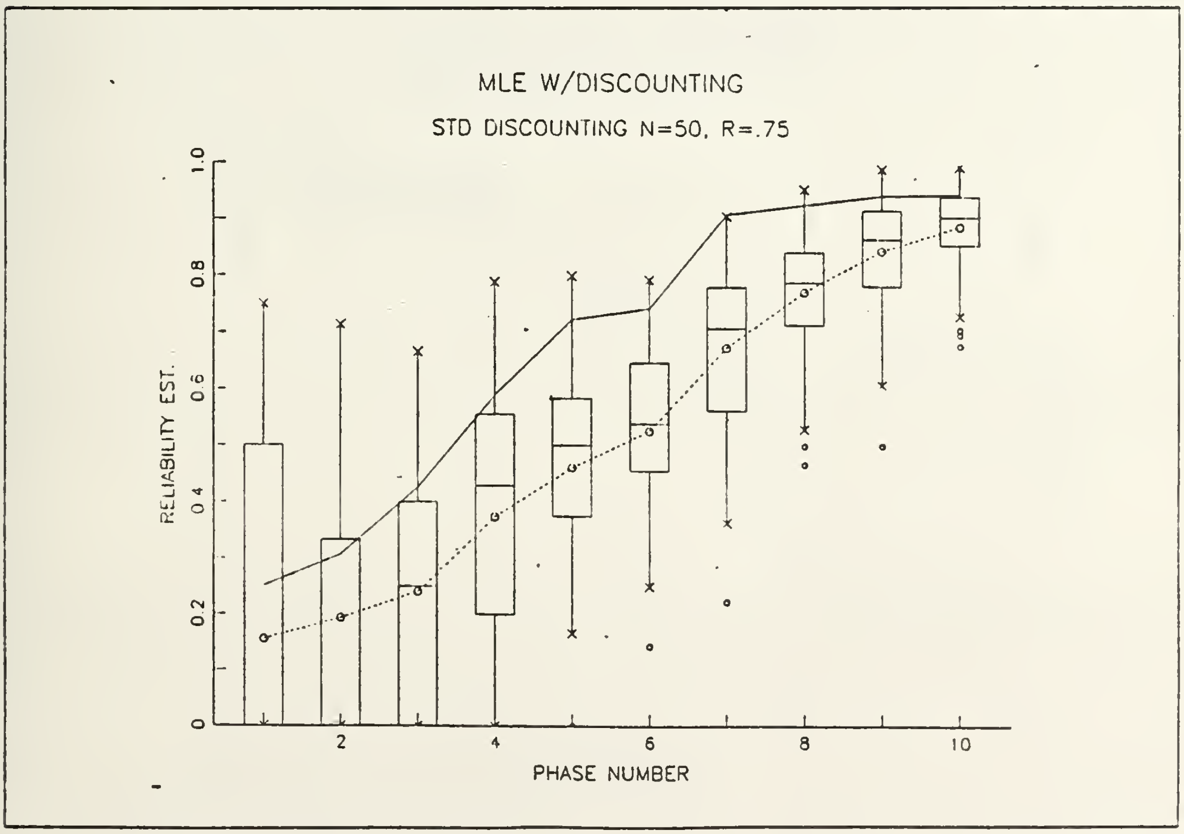
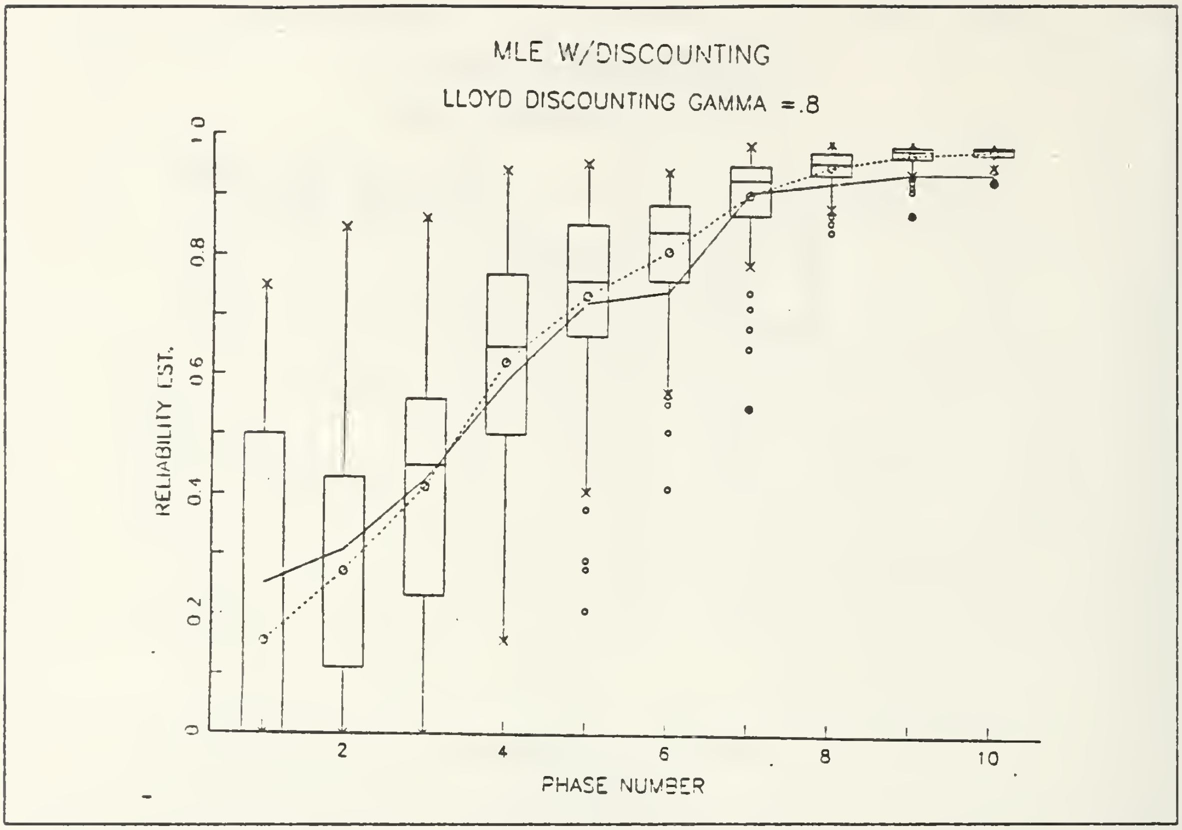
6
8
10
PHASE NUMBER


MLE W/DISCOUNTING
STD DISCOUNTING N=20, R=.75
RELIABILITY EST.
PHASE NUMBER
0
0.2
0.4
0.6
0.8
1.0
2
4
6
8
10


MLE W/DISCOUNTING
LLOYD DISCOUNTING GAMMA =.999
RELIABILITY EST.
PHASE NUMBER
0.2
0.6
0.8
2
4
6
8
10


EXPONENTIAL REGRESSION EST.
LLOYD DISCOUNTING GAMMA =.999
RELIABILITY EST.
0
0.2
0.4
0.6
0.8
1.0
2
4
6
8
10
PHASE NUMBER


MLE W/DISCOUNTING
STD DISCOUNTING N=50, R=.75
RELIABILITY EST.
0
0.2
0.4
0.6
0.8
1.0
2
4
6
8
10
PHASE NUMBER



MLE W/DISCOUNTING
LLOYD DISCOUNTING GAMMA =.8
RELIABILITY EST.
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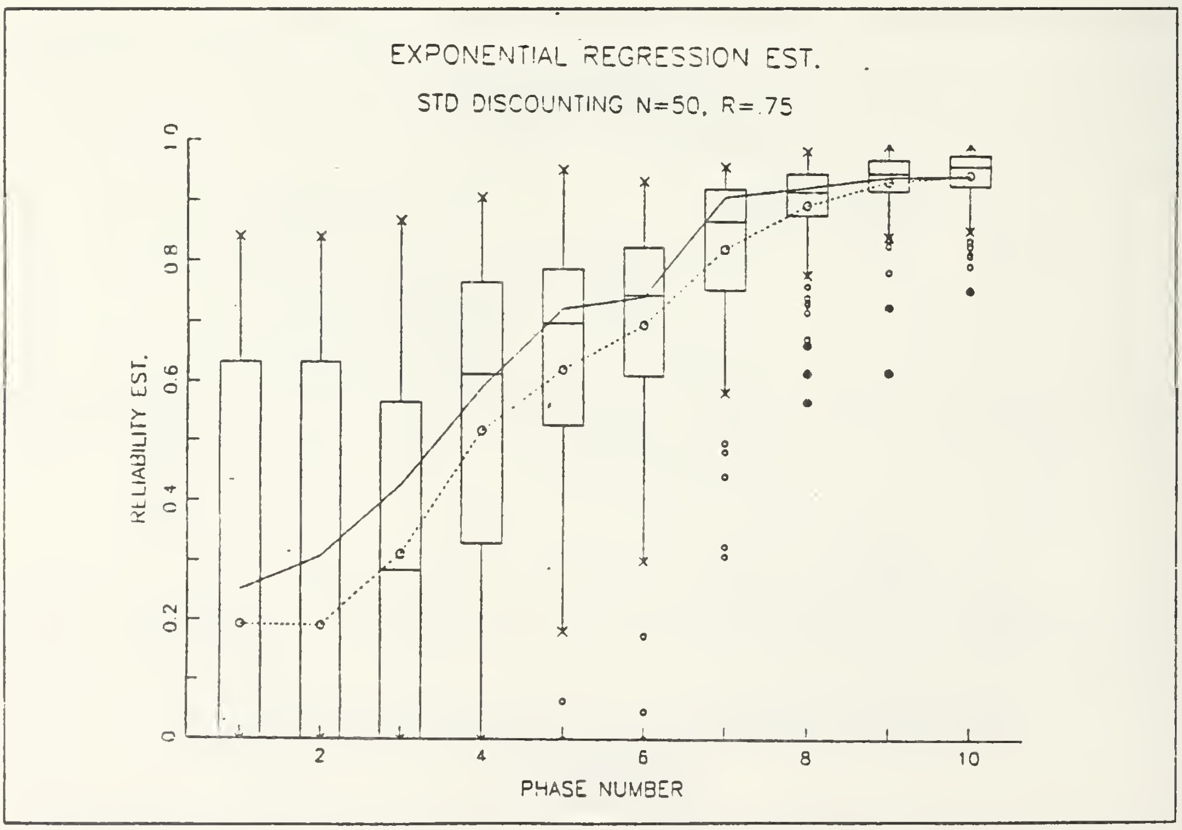
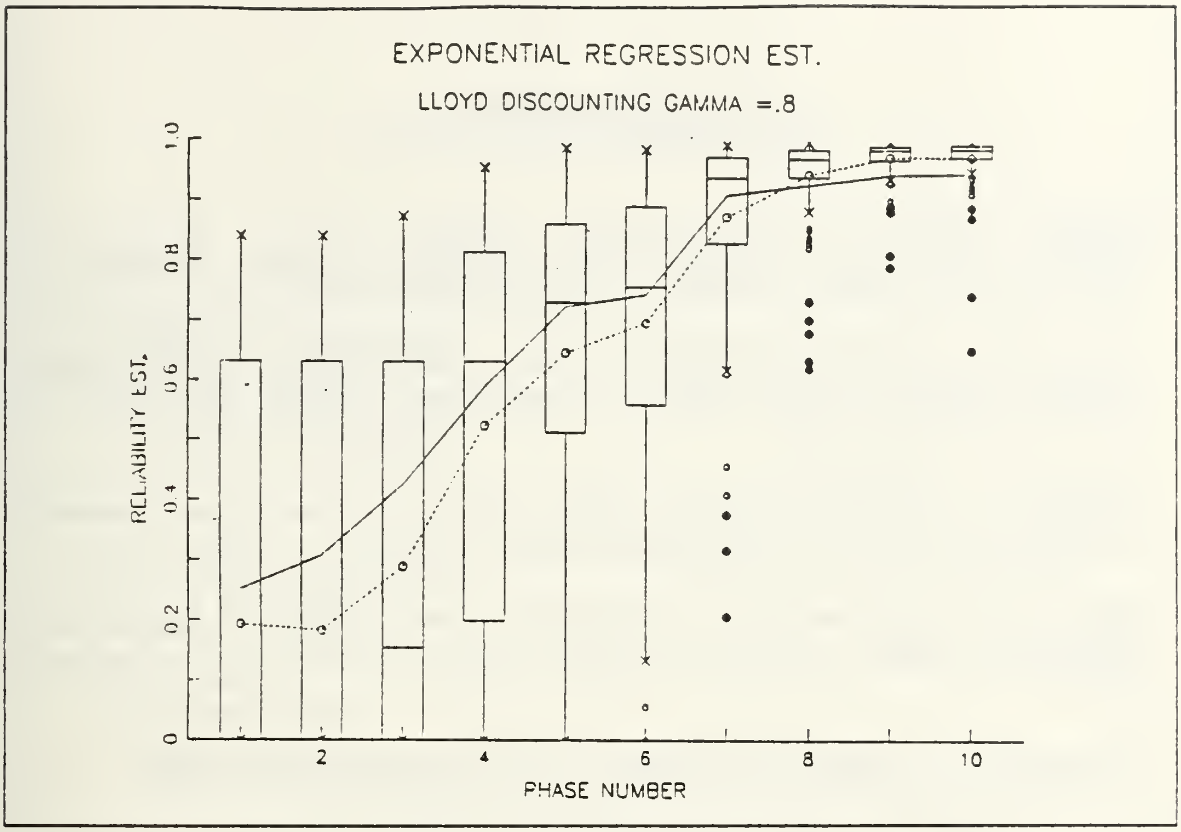
0.2
0.4
0.6
0.8
1.0
2
4
6
8
10
PHASE NUMBER



EXPONENTIAL REGRESSION EST.
STD DISCOUNTING N=50, R=.75
RELIABILITY EST.
0
0.2
0.4
0.6
0.8
1.0
2
4
6
8
10
PHASE NUMBER



EXPONENTIAL REGRESSION EST.
LLOYD DISCOUNTING GAMMA =.8
RELIABILITY EST.
0
0.2
0.4
0.6
0.8
1.0
2
4
6
8
10
PHASE NUMBER

LIST OF REFERENCES

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